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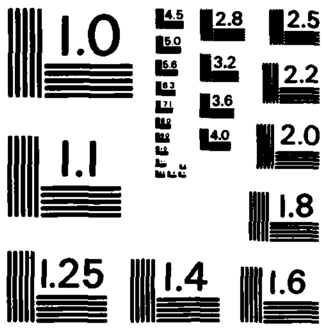

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MEMORANDUM REPORT BRL-MR-3461

APPROACH TO MODELING THE DISTRIBUTION OF THE MESSAGE SERVICE TIMES FOR THE FIRE SUPPORT TEAM EXPERIMENT

Ann E. McKaig

September 1985

US ARMY BALLISTIC RESEARCH LABORATORY
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The first experiment interfacing military personnel with the Artillery Control Environment (ACE) software was conducted 8 May - 10 June 83. This experiment demonstrated the feasibility of using the automated techniques of the Command Post Exercise Research Facility (CPXRF), located at the Human Engineering Laboratory (HEL), for fire support control experiments. The distribution of message service times was selected as one of the measures of performance (MOP) used to quantify the effects of message intensity and communication degradation

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on the Fire Support Team Headquarters' (FIST HQ) ability to perform fire support coordination. This report discusses the approach, verification, and preliminary results of modeling the distribution of FIST HQ service times of fire request (FR) messages and artillery target intelligence (ATI) messages.

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I. INTRODUCTION

During May - June 1983, the Ballistic Research Laboratory (BRL) and the Human Engineering Laboratory (HEL) conducted an experiment at the joint BRL-HEL Command Post Exercise Research Facility (CPXRF). The overall purpose of the experiment was to demonstrate the feasibility of utilizing the automated techniques of the facility to run fire support control experiments. For the first time, military players interfaced with the newly developed Artillery Control Environment (ACE) software.

To demonstrate this capability, the effects of message intensity and communication degradation on the FIST HQ ability to perform fire support coordination were studied. One of the measures of performance used to quantitatively evaluate these effects was the distribution of message service times for the fire support team. Service time has been defined to be the time required by the FIST HQ to service a message beginning at the time an acknowledgement (ACK) is sent from the fire support team digital message device (FIST DMD), indicating receipt of a message, until the time a response message is first transmitted. This measure indicates the time a message spends in the FIST DMD message queue combined with the processing and decision time of the FIST HQ.

An analysis of variance (ANOVA) procedure performed on the FIST HQ message service time data indicated that level of intensity, communication degradation level, message type, team, and replicate (of treatment combination) all had a significant effect on the FIST HQ service time.[1] Only the service time distributions of fire request (FR) and artillery target intelligence (ATI) messages were modeled and, because message type was significant, both types were modeled separately. These were the only message types (other than freetext messages which were excluded) that were not automatically forwarded by the FIST DMD while it was operating in the automatic mission mode.[2] The service times for ***fire request*** messages were modeled based on the data collected for the 36 two-hour cells (all significant factors considered). While there was marked variation among the median ATI service times by experimental condition, the service times for ***ATIs*** were combined over team and replicate and modeled based on 9 eight-hour cells. This was expedient due to a short time frame.

The model, or models, describing the distribution of FIST HQ message service times can eventually be incorporated into large simulation codes of Field Artillery command, control, and communications (C^3) and a FIST HQ simulator which is currently under development.

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[1] Smith, Jill, Grynovicki, Jock O., et al., "Fire Support Team Experiment," BRL-MR-3422, December 1984. AD A150 297

[2] After the FIST HQ had received an initial fire request and decided who would handle the mission, the fire mission was forwarded in the automatic mission mode. All subsequent messages for that mission were automatically routed through the FIST DMD. If a message was not acknowledged in four transmissions, the FIST DMD operator was notified, the message was placed in his message queue and then transmitted manually.

II. MODELING APPROACH

Identification of an appropriate functional form with which to model the FIST HQ message service time data involved at least three major steps. **First,** was the selection of a preliminary model which was based on: 1) an understanding of the origin of the data to help choose the family of functional forms to be tested, and 2) graphical representation of the data and observation of specific functions that would produce forms similar to the ones exhibited by the graphs. After the selection of a preliminary model, the **second** step was to determine the relevant model parameter values which, in this case, had to be estimated from the empirical data. The **third,** and final, step in the model identification process was the verification of the model. This was accomplished by using appropriate goodness-of-fit tests to compare the observed distributions of ATI and fire request messages with the hypothesized, or theoretical, model. For ATI messages, the chi-square (χ^2) goodness-of-fit test was used for comparison of the observed frequency distribution under varying conditions with the proposed theoretical frequency distribution. For fire request messages, due to the much smaller sample sizes and the non-normality exhibited by the observed data, a specialized Kolmogorov-Smirnov test was used to test the fit of the same model.

A. Selection of the Preliminary Model

One of the fundamental problems of modeling any type of data is the determination of the criteria to select an appropriate family of densities from which to identify a reasonable functional form to fit to the observed data. Typically, service times exhibit considerable variation and may usually be regarded as random variables. It is therefore natural to represent service times in terms of a random process. The *exponential* family of densities, frequently invoked in modeling studies to describe service time processes, was selected as the preliminary functional form for this model. This was primarily based upon the descriptive properties exhibited by the FIST HQ observed service time frequency distributions.

The observed frequency distributions of the two message types, fire request and artillery target intelligence, have been graphically represented as the histograms in the figures of Appendices A and B, respectively.* The histograms for either message type do not exhibit the property of symmetry but, in general, taper off to the right of the peak of the distribution and can be described as asymmetric.

There is a very distinct "rise and fall" pattern evident in all the artillery target intelligence message histograms; this same pattern is repeated throughout most of the histograms of the fire request messages. However, the shape of this "rise and fall" pattern of the observed frequency distributions varied not only between, but also within, the two message types. This shape is a descriptive measure of the amount of dispersion (variation) within the observed data and is indicative of how strongly each distribution

* It should be noted for the purpose of comparison that the x-axis for FRs and ATIs is scaled the same, 0.0 seconds to 50.0 seconds, while the FR y-axis (0 to 6) is scaled at one-half the ATI y-axis (0 to 12). Also, for both FRs and ATIs, whenever observations occurred at $\geq$ 50.0 seconds, they were grouped into a single cell and graphed at x=50.0 seconds. The actual range of the data within this cell has been annotated at this point.

concentrated about some particular central value, in this case, the median service time. The measure of dispersion should be large when the spread of the distribution about the median is large and should be small when the spread is negligible. Variations in shape among the distributions were not completely unexpected since there was marked heterogeneity of variance among the two-hour cells of the original observations.[1]

Although not evident from the histograms, the origin of each observed frequency distribution was translated on the X-axis from an unknown location point (threshold) to zero. This location point is defined to be the smallest observed service time for a set of responses under particular experimental conditions. The value of the location point ranged between 3.0 and 6.0 seconds for ATIs with the median and mode being 4.0 seconds, while for fire requests (FR) it ranged between 4.0 and 25.0 seconds with 8.0 seconds as both the median and mode.

The probability density function selected to initially model the FIST HQ service times had to be able to profile the descriptive properties mentioned. In summary, the distribution had to satisfy the following criteria:

1) be *asymmetrical,*

2) include a parameter, or parameters, to *measure both the dispersion and the rate at which the "tail" of the function tapered off,*

3) include a *measure of location.*

The three-parameter Weibull distribution from the exponential family of densities satisfied these criteria. If the random variable service time, call it X, has a three-parameter Weibull distribution, then its probability density function $f_X(x)$ of X has the form

$$f_X(x) = \begin{cases} \dfrac{\beta}{\alpha} \left(\dfrac{x - \nu}{\alpha}\right)^{\beta-1} \exp\left[- \left(\dfrac{x - \nu}{\alpha}\right)^{\beta} \right], & \text{if } x \geq \nu \\ 0, & \text{if } x < \nu . \end{cases} \tag{1}$$

The three constants $\beta > 0$, $\alpha > 0$, and $\nu \geq 0$ are the parameters of the distribution. The parameter β determines the shape of the density, $1/\alpha$ is a scale parameter that measures the steepness of the fall of the distribution, and the parameter ν provides information about the smallest possible value of the random variable X (service time).

The roles of β and α are illustrated in Figures 1 and 2. In Figure 1 $f_X(x)$ is graphed for $\nu = 0.0$, $\beta = 1.5$, $\alpha = 0.5$, 2.0, and in Figure 2 $f_X(x)$ is graphed for $\nu = 0.0$, $\beta = 5.0$, $\alpha = 0.5$, 2.0.

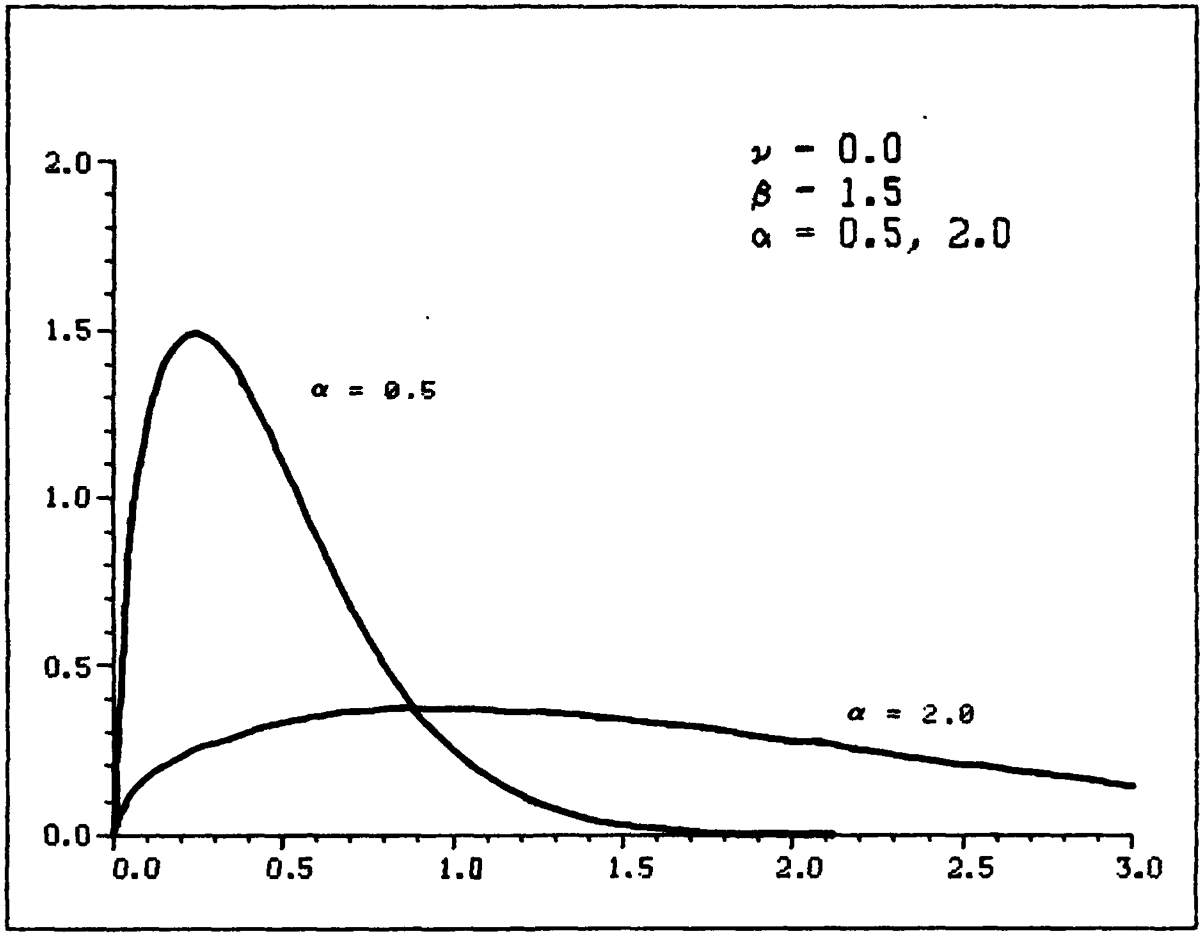

Figure 1. The Weibull Density Function for $\nu = 0.0$, $\beta = 1.5$, $\alpha = 0.5, 2.0$

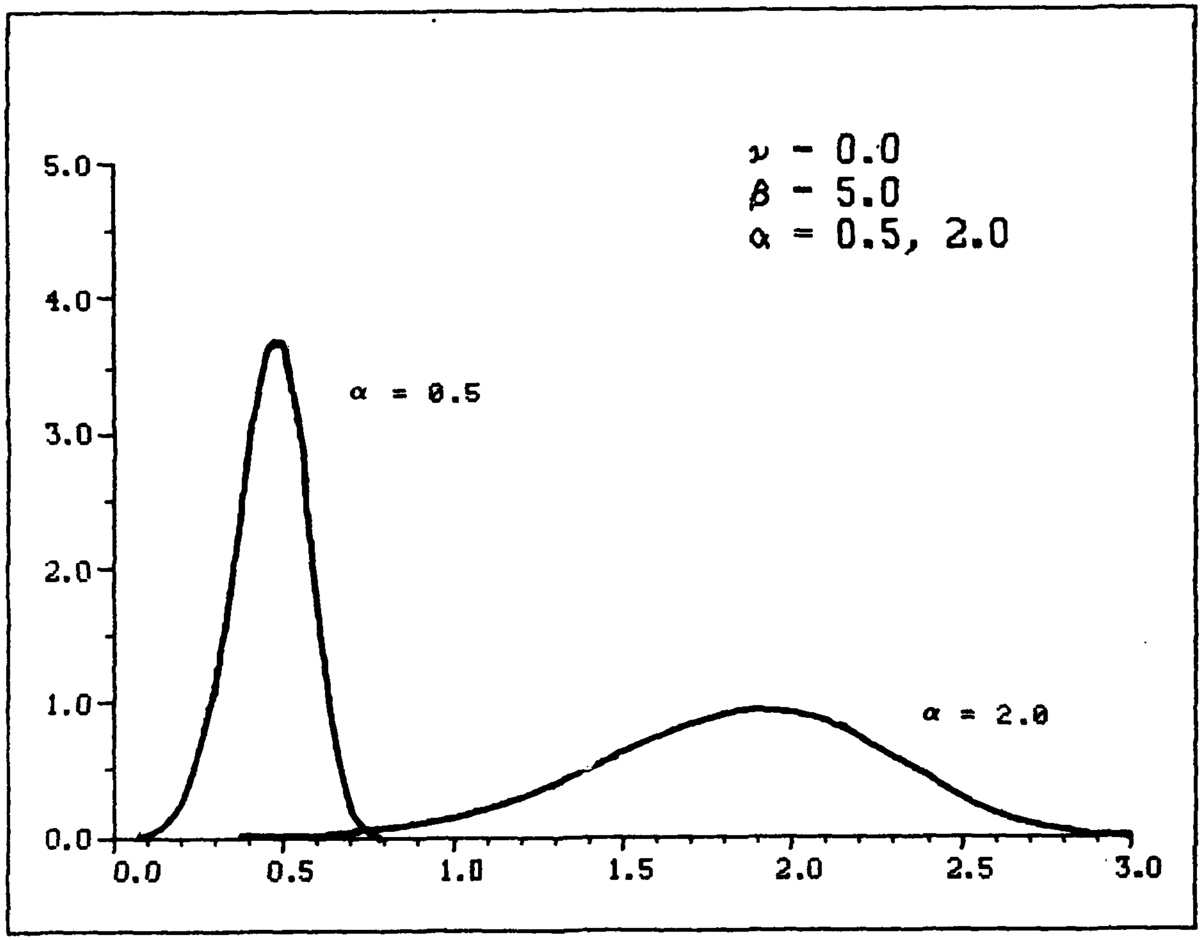

Figure 2. The Weibull Density Function for $\nu = 0.0$, $\beta = 5.0$, $\alpha = 0.5, 2.0$.

B. Parameter Estimation for the Weibull Distribution

Once the preliminary model for the FIST HQ service time data had been selected, the next step was to determine the relevant parameter values. Recall that the parameters β, α, and ν of the Weibull distribution denote shape, scale and location, respectively.

Analytical estimates of the three distribution parameters, when all are unknown, should be obtained iteratively for which the method of maximum likelihood is appropriate. Since the maximum likelihood estimators cannot be obtained explicitly, another method proposed by Dubey[2,3] provides a simple point estimator for ν and also allows one to explicitly obtain percentile estimators for β and α. These estimates may be considered satisfactory approximations of β, α, and ν to be used as initial guesses in the solution of the likelihood equations.

Suppose that $x_{(1)}, x_{(2)}, \cdots, x_{(n)}$ represent n ordered observations of a random sample of the FIST HQ service times from the Weibull population with $f_X(x)$ given by Eq. (1). Then Dubey proposed that a simple estimator for ν is of the form

$$\hat{\nu} = \frac{x_{(1)}\, x_{(k)} - x^2_{(j)}}{x_{(1)} + x_{(k)} - 2x_{(j)}}\,, \tag{2}$$

where $x_{(j)}$ and $x_{(k)}$ are any of the ordered observations such that $x_{(1)} < x_{(j)} < (x_{(1)}\, x_{(k)})^2$. Throughout the process of computing the point estimates of ν for each cell modeled, $x_{(j)}$ and $x_{(k)}$ are selected as the ordered observations corresponding to the 25th and 75th sample percentiles, respectively.

Once ν has been estimated, a percentile estimator of the shape parameter, β, can be explicitly obtained based on any two sample percentiles. However, Dubey[3] showed that the 17th and 97th sample percentiles asymptotically yield the best percentile estimator of β. These are the sample percentiles used in the estimation of the shape parameter. The location parameter ν is neither known to be 0 nor has it yet been set equal to 0 by an appropriate transformation of the data. A slight modification of the percentile estimator of β, based on two ordered sample observations from the Weibull population and assuming ν is known but different from 0, is given by:

$$\hat{\beta} = \frac{\ln(-\ln(1-p_1)) - \ln(-\ln(1-p_2))}{\ln(y_{p_1} - \hat{\nu}) - \ln(y_{p_2} - \hat{\nu})}\,, \tag{3}$$

where

$$\begin{aligned} &0 < p_1 < p_2 < 1;\ p_1 = .1673,\ p_2 = .9737, \\ &y_{p_i} = y_i, \quad i = 1,2 \end{aligned} \tag{4}$$

and

[2] Dubey, Satya D., "Hyper-efficient Estimator of the Location Parameter of the Weibull Laws," Naval Research Logistics Quarterly, Vol. 13, 1966.

[3] Dubey, Satya D., "Some Percentile Estimators for Weibull Parameters," Technometrics, Vol. 9, 1967.

y_p, the 100 p per cent percentile of the sample, is defined as

$$y_p = \begin{cases} x_{(np)} & \text{if np is an integer} \\ x_{([np]+1)} & \text{if np is not an integer} \end{cases} \tag{5}$$

where [np] denotes the largest integer not exceeding np and, as previously stated, n is the size of the sample.

Finally, when it is assumed that the shape parameter is known and, again, that ν is known but different from 0, then a modified single observation best percentile estimator of α is

$$\hat{\alpha} = y \,/\, \ln^{1/\hat{\beta}} (1 - p)^{-1} , \tag{6}$$

where y is any sample percentile corresponding to the cumulative probability p ($0 < p < 1$). For the sake of consistency, p was set equal to p_1 and y set equal to y_{p_1}, where p_1 and y_{p_1} were previously defined for the development of $\hat{\beta}$.

The point estimates of $\hat{\beta}$, $\hat{\alpha}$, and $\hat{\nu}$, calculated using the above procedures, are given in Tables C-1A thru C-3A, and C-10A for fire requests and ATIs, respectively. These estimates were input as initial guesses to the Biomedical Computer Programs P-Series (BMDP) maximum likelihood estimation program with one exception. The point estimates of the location parameter, $\hat{\nu}$, for both FRs and ATIs closely approximated the minimum service time observed in each of the cells being modeled. Since the location parameter has been defined as the smallest order statistic of an ordered set of observations, and no prior knowledge existed to support the theory that ν should assume some particular value ν_0, it was decided to fix each estimate of $\hat{\nu}$ as the smallest observed service time within each cell. These fixed estimates of the location parameter, along with the maximum likelihood estimates of $\hat{\beta}$ and $\hat{\alpha}$, are given in Tables C-1B thru C-3B for fire requests and in Table C-10B for artillery target intelligence messages.

1. Fire Request Messages

The maximum likelihood estimates of both $\hat{\beta}$ and $\hat{\alpha}$ for fire request messages (see Tables C-1B thru C-3B) proved to be quite varied within each of the significant experimental conditions: level of communication degradation, replicate, team, and level of intensity. Large values of 8.844 and 35.391 for $\hat{\beta}$ and $\hat{\alpha}$, respectively, occurred at 30% communication degradation, replicate 1, team 3, and low intensity. A possible explanation of why such large estimates occurred during low intensity will be given during the description of the parameter estimates of $\hat{\nu}$.

With the value of 35.391 excluded, the maximum likelihood estimates of $\hat{\alpha}$ appeared to be fairly consistent across the factors, ranging from a low of approximately 1.000 to a high of less than 15.000. It should be noticed in the tables that small values of $\hat{\alpha}$ and $\hat{\beta}$ are usually paired and if $\hat{\alpha}$ is large, then $\hat{\beta}$ is also large. Recall that these two parameters affect the shape of the distribution (i.e., how highly concentrated the distribution is about a selected measure of central tendency). The variance of the

random variable, FIST HQ fire request message service time, provides information about the "scatter" or amount of concentration of the observations. Since the variance of a Weibull distribution is dependent upon the two parameters α and β, the wide range of values for the estimates of $\hat{\alpha}$ and $\hat{\beta}$ is consistent with the non-homogeneity of variance of the observed service times mentioned earlier.

The parameter ν serves only as a measure of location and does not influence the shape of the distribution in any way. Tables C-1B thru C-3B give the maximum likelihood estimates of $\hat{\nu}$ for the fire request service time data. Of the four factors involved, the biggest difference among the estimates of $\hat{\nu}$ occurred between the two teams. Estimates of $\hat{\nu}$ ranged from 8.000 to 25.000 seconds for team 3 and 4.000 and 12.000 seconds for team 4. There were two extreme values for $\hat{\nu}$ of 25.000 seconds during Rep (replicate) I and 16.000 seconds during Rep II for team 3 while operating under low intensity and 30% communication degradation. It should be pointed out that as the level of intensity increased from low to high, $\hat{\nu}$ slightly decreased. At low intensity $\hat{\nu}$ ranged between 5.000 and 25.000 seconds, at medium intensity it ranged from 5.000 to 14.000 seconds, and at high intensity $\hat{\nu}$ ranged from 4.000 to 12.000 seconds. This may appear to be an anomolous situation, but often a server (in this case the FIST HQ) increases his effort when under pressure, perhaps due to a long message queue.

With the two large estimates of $\hat{\nu}$ for team 3 removed, the mean of $\hat{\nu}$ with all factors considered is 8.412 seconds. The mean $\hat{\nu}$ over all associated levels of each experimental condition proved to be quite stable ranging from 8.361 seconds (over levels of intensity) to a high of only 8.524 seconds (across teams). These estimates may provide some insight, particularly for modeling purposes, as to the minimum amount of time required by the FIST HQ to review and transmit a fire request message; this time would also include the time the message remained in the FIST DMD message queue. They may help in laying a theoretical foundation upon which ν can be fixed at some particular ν_o.

2. Artillery Target Intelligence Messages

Maximum likelihood estimates of $\hat{\beta}$ for ATIs (see Table C-10B) ranged between 1.001 and 2.075 with 8 of the 9 estimates either greater than or equal to 1.000 but less than 2.000. When $\hat{\beta}$ is estimated to be very close to 1.000, the Weibull distribution has no mode and the probability density function decreases monotically as the random variable service time increases. Where $\hat{\beta}$ is distinctly greater than 1.000, such as for low intensity and 00% communication degradation, the distribution is unimodal.

With the exception of the extremely large value of $\hat{\alpha}$, 12.555, at high intensity and a 30% communication degradation level, the $\hat{\alpha}$ estimates exhibited only a slight degree of variation. Excluding the $\hat{\alpha}$ value of 12.555, $\hat{\alpha}$ ranged between 4.837 and 9.452. The large estimate of $\hat{\alpha}$ at high intensity and 30% communication degradation is not really surprising as it is indicative of a much longer tailed distribution (larger service times) which is not as concentrated about the large median service time of 19.5 seconds for this cell.[1] This is consistent with the fact that when the FIST HQ is operating under such

severe conditions, FRs have a greater priority and will be processed before any incoming ATIs which are then forced to remain in the message queue, resulting in a more widespread distribution of service times for this particular cell than any other. This is evident from its histogram of observed service times (see Appendix B) which range between 0 and 93 seconds, a range which is almost half again as large as the next largest range of service times occurring at low intensity and 30% communication degradation.

Recall that $\hat{\nu}$ was fixed as the shortest observed time in which a message could be processed and transmitted. For ATIs there was very little variation among the estimates either within the levels of intensity or communication degradation; overall, $\hat{\nu}$ varied between a low of 3.000 seconds and a high of only 6.000 seconds. The largest observed value of $\hat{\nu}$, 6.000 seconds, occurred at high intensity and 30% communication degradation, probably for the same reason as mentioned in the previous paragraph.

C. Transformation of the Weibull Distribution to the Exponential Distribution

The estimates $\hat{\beta}$, $\hat{\alpha}$, and $\hat{\nu}$, of the three-parameter Weibull distribution provide a reasonable analytical interpretation of the effects that various combinations of level of intensity and level of communication degradation had on the distribution of the FIST HQ service times for fire request and artillery target intelligence messages. However, considerable effort is required to compute these estimates regardless of the procedure utilized.

The exponential distribution was initially ruled out because this distribution is limited by the assumption of a constant service rate (the function is monotonically decreasing) while the Weibull distribution can be written to include variable (increasing and decreasing) service rates. However, an exponential model can be used in situations where the service rate is not constant as long as an appropriate transformation of the Weibull distributed random variate, FIST HQ service time, is made (see Appendix D).

To make such a transformation, the following assumptions about the three parameters of the Weibull distribution had to be made: 1) the parameters β and ν are known while α is assumed unknown, 2) $\beta = 1.000$, and 3) $\nu = 0.000$. In Section II-B, it was shown that β was equal to or "near" 1.000 about 90% of the time for ATIs, and estimated at or "near" 1.000 about 67% of the time for FRs. It therefore seemed reasonable to conclude that the shape parameter β was indeed known and could be set equal to 1.000. Based on this conclusion an exponential probability density function with both scale and location parameters was derived. To obtain the final, single parameter exponential model each observed service time within a particular cell was adjusted by an amount equivalent to $X_{min} = \nu$ for that particular cell. Values of $X_{min} = \nu$ for the cells are given in Tables C-1B, C-2B, and C-3B for FRs and Table C-10B for ATIs.* This translation effectively shifted the origin of the observed service time distributions to $X_{min} = \nu = 0.000$.

* Note that these are the same values of ν that were fixed during the computation of maximum likelihood estimates of α and β for the Weibull distribution.

Since the assumptions have been met, the three parameter Weibull distribution is equivalent to an exponential distribution with scale parameter $\theta = 1/\alpha$. The probability density function of the new random variate Y, adjusted FIST HQ service time, becomes

$$f_Y(y) = \begin{cases} \theta\, e^{-\theta y}, & \text{if } y \geq 0, \\ 0, & \text{if } y < 0. \end{cases} \tag{7}$$

Maximum likelihood estimates of θ for varying communications degradation are presented in Tables C-4 thru C-6 and C-11 (Appendix C) for fire request messages and ATIs, respectively. Since the scale parameter α of the Weibull distribution was assumed unknown, the maximum likelihood estimates of θ were not computed as $1/\alpha$ but rather as $[\,1.0/\,(\sum_{i=1}^{N} y_i/N)\,]$, where $y_i = x_i - \nu$ and $(\sum_{i=1}^{N} y_i/N)$ is the mean of a cell of adjusted service time observations. There is, however, reasonable agreement between the maximum likelihood estimates of θ as generated by the above method and likelihood estimates of θ if they had, in fact, been computed as $1/\alpha$. As an example, in Table C-10B for ATIs, with 00% communication degradation and low intensity, the maximum likelihood estimate of α is 6.582 making the estimate of $\theta = 1/\alpha = .152$. Table C-11 gives the maximum likelihood estimate of θ, under the same conditions, as .150.

Exponential fits to the observed data sets using the maximum likelihood estimates of θ with ν fixed at 0.000 are presented in Appendices A and B as the heavily shaded curves.

D. Model Verification

The final step in the model identification process was the verification of the single parameter exponential model. This was accomplished by performing an appropriate goodness-of-fit test on the *observed* data for each message type to compare its *hypothesized* probability distribution with the frequency distribution. The null hypothesis for the goodness-of-fit test is a statement about the identity of the probability distribution that fits the FIST HQ service time data. Normally, this hypothesized distribution is completely specified, including all parameter values. However, as previously mentioned, the unknown parameter of the exponential distribution had to be estimated from the data before any goodness-of-fit test could be carried out. It should be understood that rejection of the null hypothesis for any particular case gives no information about what the true service time population form is, only what it is not. Thus, the goodness-of-fit tests were utilized to obtain statistical support for the theory that the single parameter exponential distribution fits both the FR and ATI message service time data by acceptance of the null hypothesis.

Two well-known statistical tests for goodness-of-fit were used for the model verification process. An adaptation of the **Kolmogorov-Smirnov** (K-S) test statistic was used to verify the model for **fire request** messages, while the **chi-square** test was used to verify the same model for **artillery target intelligence** messages. Both tests are based on a comparison between the distribution of the observed sample data and the

theoretical distribution that has been hypothesized; however, the two tests use a different basis for comparison.

The technique used for testing the goodness-of-fit between the observed sets of FIST HQ service times for fire request messages and the hypothesized exponential model was the Kolmogorov-Smirnov test. Its measure of incompatibility is based on the vertical deviations between the observed and the hypothesized cumulative distribution functions. The null hypothesis of the K-S test specifies that for the observed random variable, FR service time, the cumulative probability function is some $F_0(x)$. This is tested against the alternative hypothesis that the cumulative probability function of the observed random variable is different from $F_0(x)$. This null hypothesis was tested at a level of significance of 0.05. One important assumption of the K-S test is that the sets of observations come from a completely specified continuous distribution. However, the scale parameter θ of the exponential distribution was not known and had to be estimated from the sample data, hence, the standard K-S test no longer applies because the commonly tabulated critical values of the test statistic become conservative (although exactly how conservative is generally unknown). In this case, tables listing critical values for a specialized Kolmogorov-Smirnov test, for the exponential distribution when its population mean is unknown, had to be used in place of the standard tables.[4] This specialized test provides a goodness-of-fit test which can be used with sample sizes that are considered too small for use with the chi-square test.

The Kolmogorov-Smirnov goodness-of-fit test statistic measures agreement as the absolute value of the largest vertical difference between the graphs of the two cumulative relative frequency distributions. It is defined as $D = \max_x | F_0(x) - S(x) |$, where $S(x)$ is the sample (observed) cumulative distribution function, $F_0(x)$ is the hypothesized cumulative exponential distribution function with parameter θ, and the max over x is the largest of the differences $| F_0(x) - S(x) |$ in the neighborhood of each observed value of x. Since the calculated value of D is a measure of agreement between the two distributions, a large value for D tends to discredit the proposed null hypothesis.

The computed values of the test statistic D are given in Tables C-7 thru C-9. Under 00% communication degradation (see Table C-7), over all levels of intensity, the computed K-S values show that 58% of the time the null hypothesis could not be rejected. Table C-8 shows that the best results were obtained at the 15% communication degradation level where 9 of 12 cells passed the goodness-of-fit test when the method of maximum likelihood was used to estimate θ. Finally, at the 30% communication degradation level, 8 of the 12 cells passed using maximum likelihood estimates of θ.

Over all three levels of communication degradation, the null hypothesis could not be rejected at the 0.05 level of significance for 24 of the 36 cells tested (67%) when $F_0(x)$ was computed using maximum likelihood estimates of θ.

[4] Lilliefors, H. W., "On the Kolmogorov-Smirnov Test for the Exponential Distribution with Mean Unknown," Journal of the American Statistical Association, Vol 64, 1969.

Based on the above results, the specialized K-S goodness-of-fit test appears to have qualified the exponential model as an appropriate fit to the FIST HQ service time data for fire request messages.

Unlike the K-S test, the chi-square test's measure of incompatibility is based simply on the vertical deviations between the observed frequencies and the associated set of expected, or theoretical, frequencies derived under the assumption that the null hypothesis is true. Strictly speaking, the chi-square test is not appropriate as a goodness-of-fit test for the FIST HQ artillery target intelligence message service time data since the exponential distribution is a continuous rather than a discrete distribution; however, it has been justified by considering the ATI service time data as grouped into a finite number of mutually exclusive and exhaustive intervals. The observations in each eight-hour cell were grouped into r, arbitrarily chosen,non-overlapping numerical categories. The data was analyzed in the form of count data, where each count represents the number of observations classified in each of the r categories.

The r categories were chosen based upon a frequently used rule concerning the size of the expected frequencies. The rule states that no more than 20% of the expected frequencies should be less than 5.0, and none can be less than 1.0. Based on this rule, the low, medium and high intensity levels for ATI messages were grouped into 8, 7, and 4 categories, respectively.

The null hypothesis to be tested is that the probability density function of the observed random variable, ATI message service time, is some $f_0(x)$, set against the alternative hypothesis that the probability density function of the observed random variable is different from $f_0(x)$.

The test statistic computed to make the aforementioned comparison is denoted as χ^2. This statistic may be defined as

$$\chi^2 = \sum_{i=1}^{r} \frac{(f_0 - f_i)^2}{f_i} \tag{8}$$

where f_0 = an observed frequency, and f_i = an expected, or theoretical, frequency for a particular category. Associated with this statistic is a number of degrees of freedom which is equal to the number of categories for each communication degradation level minus one. In addition, for this analysis, the single parameter of the exponential distribution that was estimated from the data prior to determining the expected frequencies also had to be subtracted off the total number of degrees of freedom.

From Eq. (8) it can be seen that if every observed frequency is exactly equal to the corresponding expected frequency, then the value of χ^2 is 0.0 (the smallest possible value of χ^2). The larger the discrepancies between the observed and expected frequencies, the larger the value of χ^2. The null hypothesis was tested at the 0.05 level of significance. This means that if the null hypothesis were true, the probability of observing a computed χ^2 value greater than the critical (tabulated) χ^2 value would be 0.05. The computed χ^2 values are given in the body of Table C-12. The appropriate critical χ^2

values are 12.592, 11.070, and 5.991 for 6, 5, and 2 degrees of freedom, respectively. Approximately 78% (7 of 9 cells) of the time the null hypothesis could not be rejected when maximum likelihood estimates of θ were used in computing the expected frequencies needed for evaluating the test statistic. The two conditions under which the χ^2 statistic was significant were "perfect" communications (00% communication degradation) with low intensity and 15% communication degradation with medium intensity. These large computed χ^2 values do not appear to be due to anomalies within the observed data, but rather to the unfortunate instability of the chi-square statistic since its value is affected by the number of categories within a cell and the width of each category.

The hypothesis that the population probability distribution of ATI message service times is exponential was tested and, overall, could not be rejected. Thus, at the 0.05 level of significance, the exponential distribution can reasonably be used as a model for FIST HQ service time for artillery target intelligence messages.

III. MODELING RESULTS

Modeling efforts of the FIST HQ fire request and artillery target intelligence message service time data collected during the first CPX Research Facility experiment indicate that the distribution of the service times of these message types can be initially fit with a three-parameter *Weibull distribution*. Under the assumptions that one, the shape parameter, β, of the distribution is equal (or close) to 1.000, and two, the location parameter, ν, takes on the value of the smallest order statistic of each of the data sets modeled, the distribution of FIST HQ service times for FR and ATI messages is easily transformed into a single parameter *exponential distribution*. The maximum likelihood estimate of the parameter θ of this distribution is easily computed as $\theta = [\, 1.0/\, (\sum_{i=1}^{N} y_i/N)\,]$, where the y_i are the adjusted service times and N is the sample size of the data set.

There are certain trade-offs made by electing to use the exponential distribution rather than the three-parameter Weibull distribution to model FR and ATI message service times. The Weibull distribution performed well in describing the rise-and-fall pattern exhibited by many of the data sets, accounting for both increasing and decreasing service times. However, it is considered a "short tailed" distribution and, as such, did not do well in modeling extreme value observations. On the other hand, the exponential distribution is strictly a monotonically decreasing function and assumes a constant service time rate. Although the exponential density function fell off rapidly as service time increased, in most cases, the steepness of the fall was very gradual as evidenced by the small values of θ (all $<$ 1.000) that were computed. This long tail was able to accommodate the large service times observed in some cells.

A specialized Kolmogorov-Smirnov test and the chi-square goodness-of-fit test were used to compare the distribution of the transformed service times with the theoretical, or hypothesized, distribution (i.e., the single parameter exponential distribution) for fire

request and ATI messages, respectively. For each of the cells modeled, the unknown parameter θ had to be estimated from the data before the tests could be performed. Results showed that when utilizing maximum likelihood estimates of θ, 67% of the fire request message cells (two-hour) could not be rejected at the 0.05 level of significance, while 78% of the ATI message cells (four-hour) could not be rejected at the same level of significance.

While a classical parametric model provided a reasonable fit to the distribution of the observed FIST HQ service times for FRs and ATIs, it should also be realized that it does obscure situations that can arise at the FIST HQ and ultimately influence its message service time. Problems, such as: the prioritization of particular types of missions (like Copperhead), the limited queue size of the FIST DMD (it can accommodate only 16 messages at any one time), the fact that messages may be turned away, and a varying message arrival rate at the FIST HQ (due to a change in the level of intensity of the battle) could all be handled by the development of an efficient queueing system model for the FIST HQ service time.

IV. CONCLUSIONS

A discussion of the procedure by which the FIST HQ service time data for FRs and ATIs was modeled and the techniques utilized to test the fit of the hypothesized model has been provided. It was found that while the FIST HQ was operating under the forward observer (FO) *review* control mode, the service times for fire request messages and artillery target intelligence messages were reasonably modeled by a single parameter exponential distribution.

The exponential distribution generated from the data is easy to work with and it has shown its capability of accounting for differences among levels of intensity and communication degradation, team, and replicate. However, it does "mask" other conditions existent at the FIST HQ which could also significantly influence message service time. Despite this fact, the assumption that message service times at the FIST HQ are exponentially distributed can be successfully used as input into the development of a queueing system model that would more thoroughly represent the conditions that influence the FIST HQ message service time.

ACKNOWLEDGEMENTS

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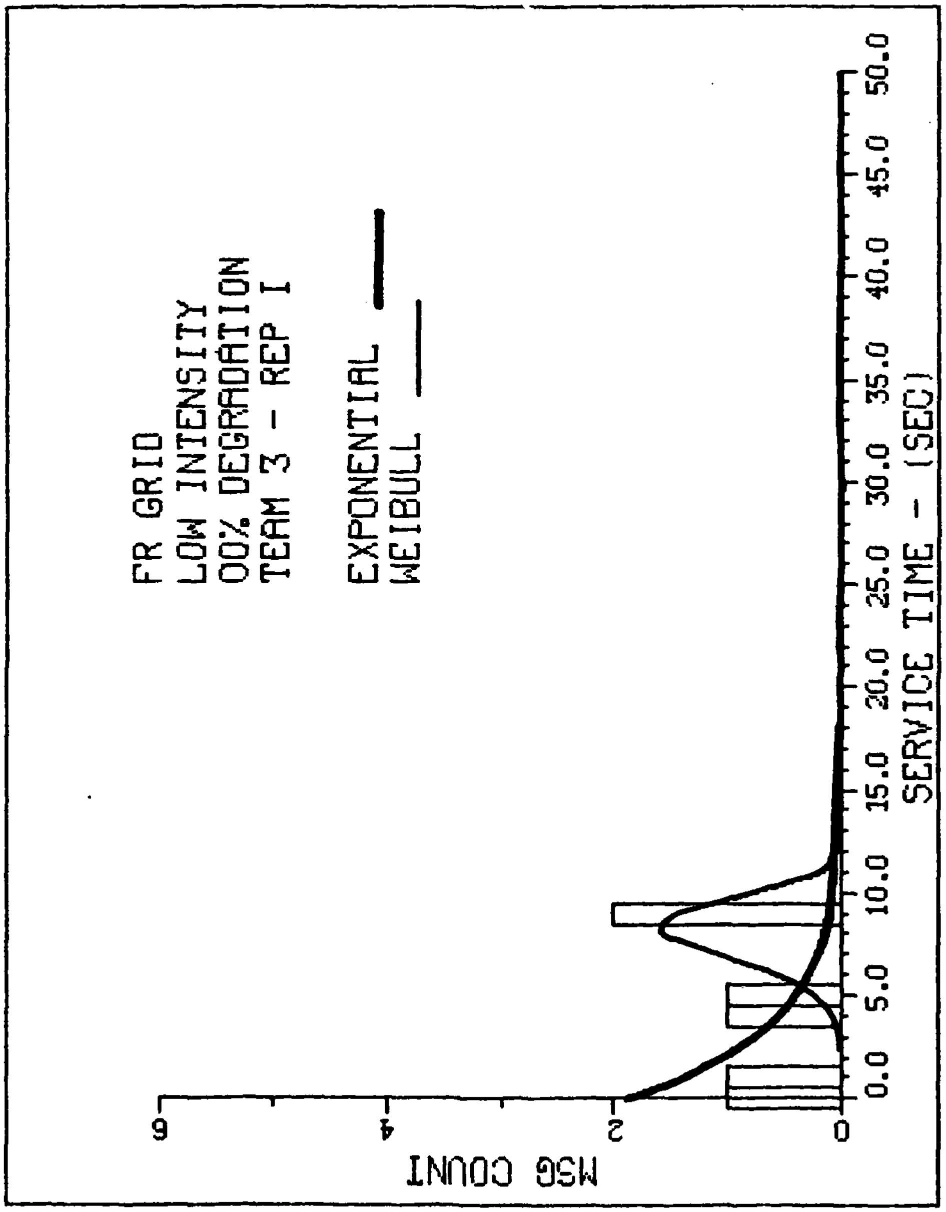
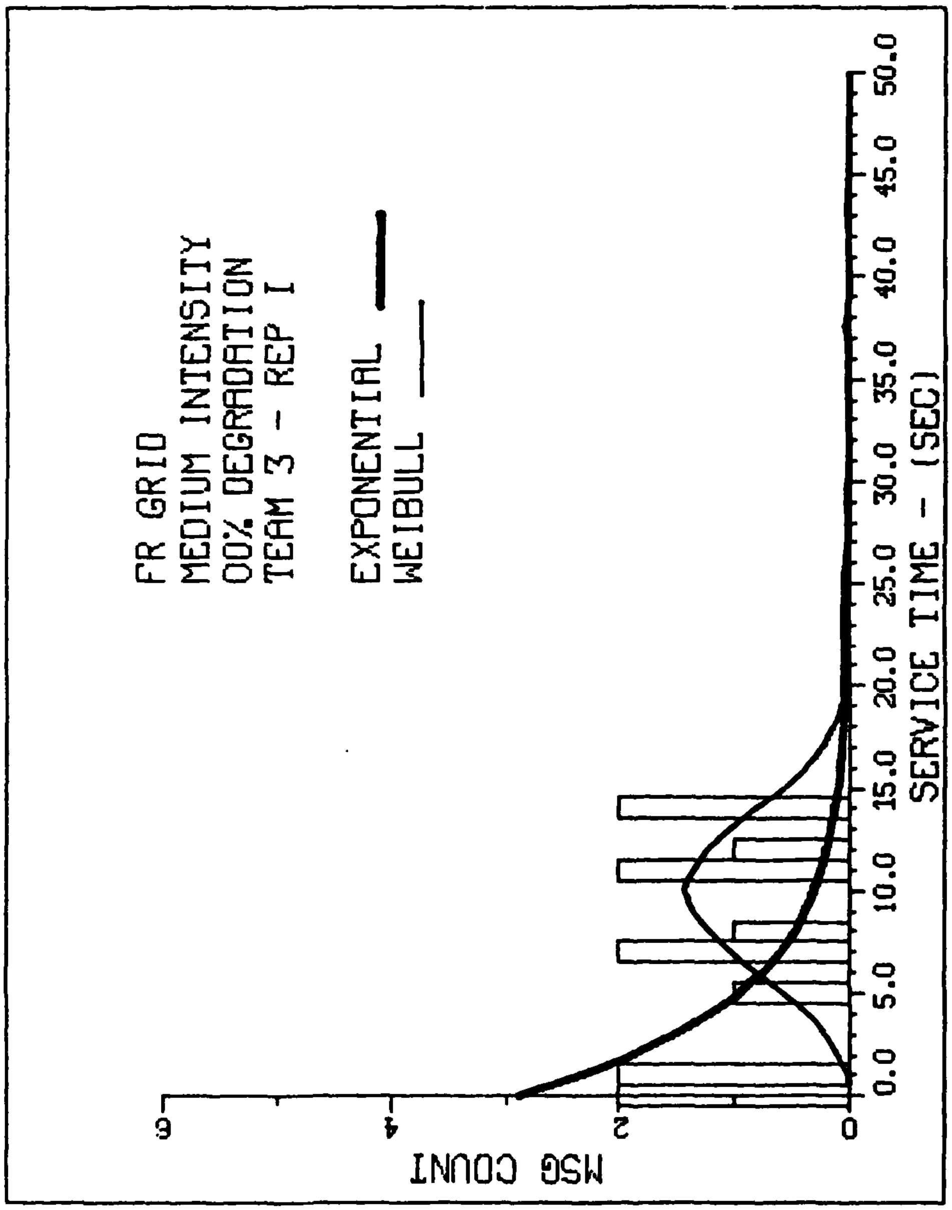
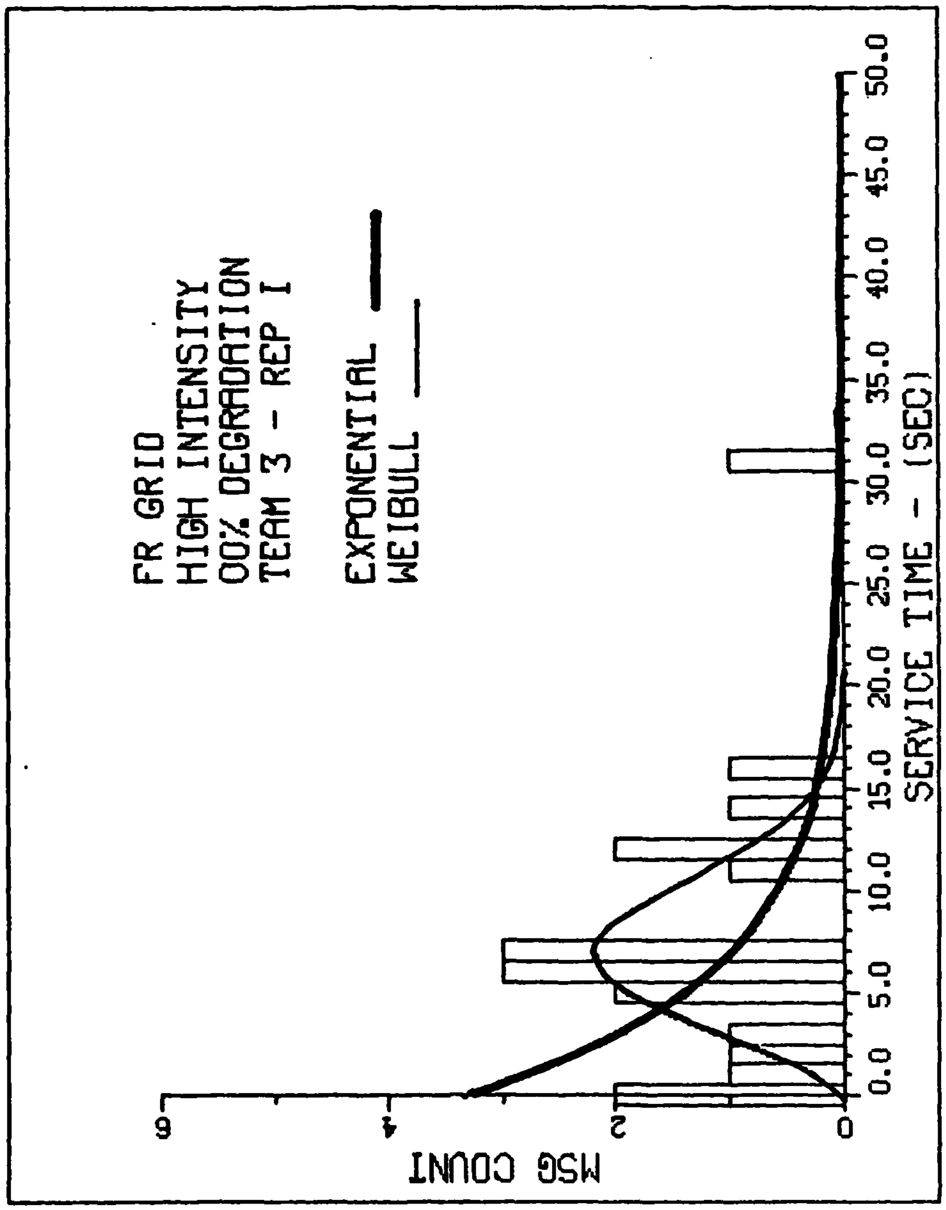
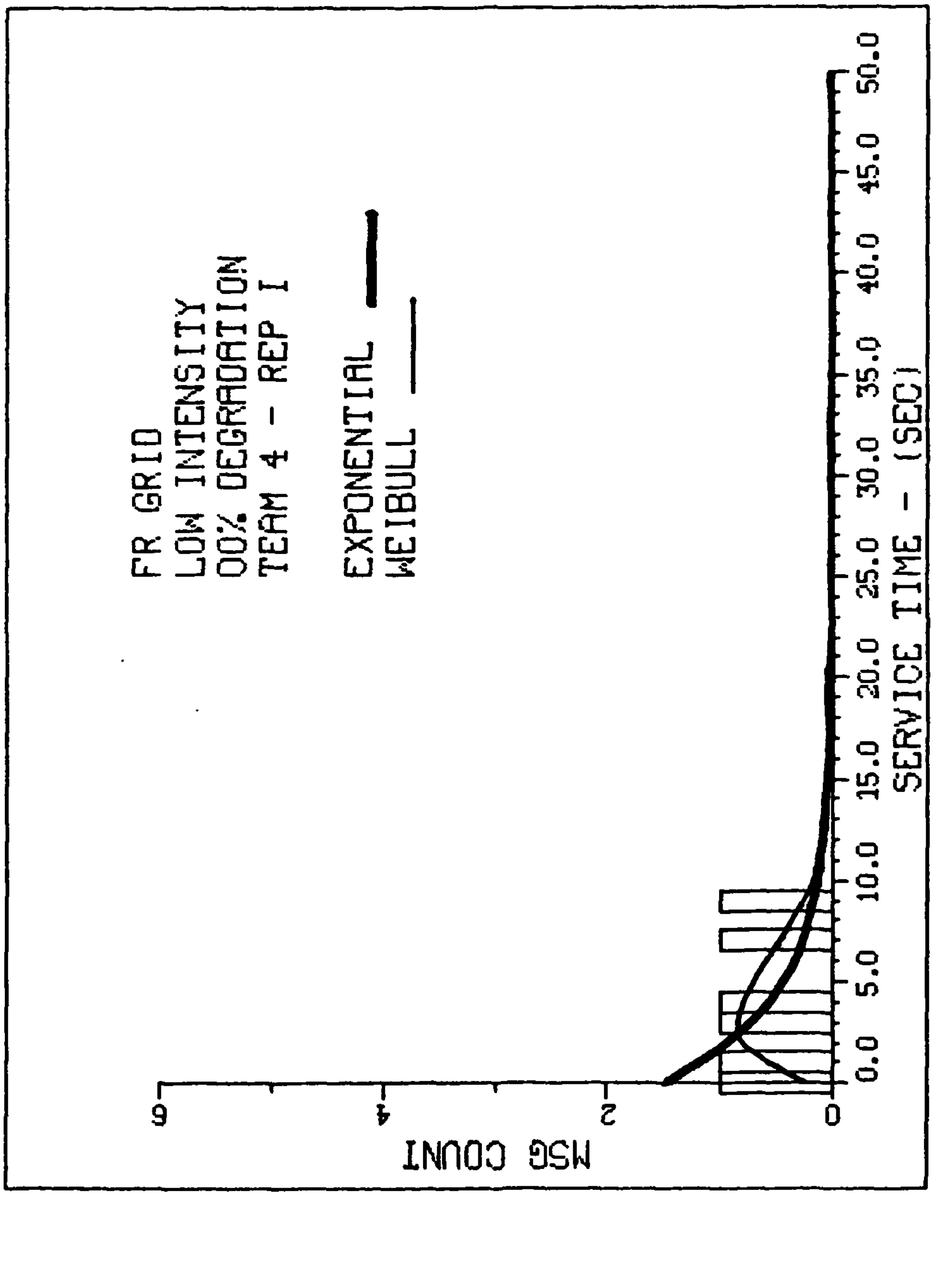
APPENDIX A.

OBSERVED AND THEORETICAL FREQUENCY DISTRIBUTIONS FOR FIRE REQUEST MESSAGES BY COMMUNICATION DEGRADATION LEVEL, REPLICATE, TEAM, AND LEVEL OF INTENSITY


FR GRID
LOW INTENSITY
00% DEGRADATION
TEAM 3 - REP I
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
SERVICE TIME - (SEC)


FR GRID
MEDIUM INTENSITY
00% DEGRADATION
TEAM 3 - REP I
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
SERVICE TIME - (SEC)


FR GRID
HIGH INTENSITY
00% DEGRADATION
TEAM 3 - REP I
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
SERVICE TIME - (SEC)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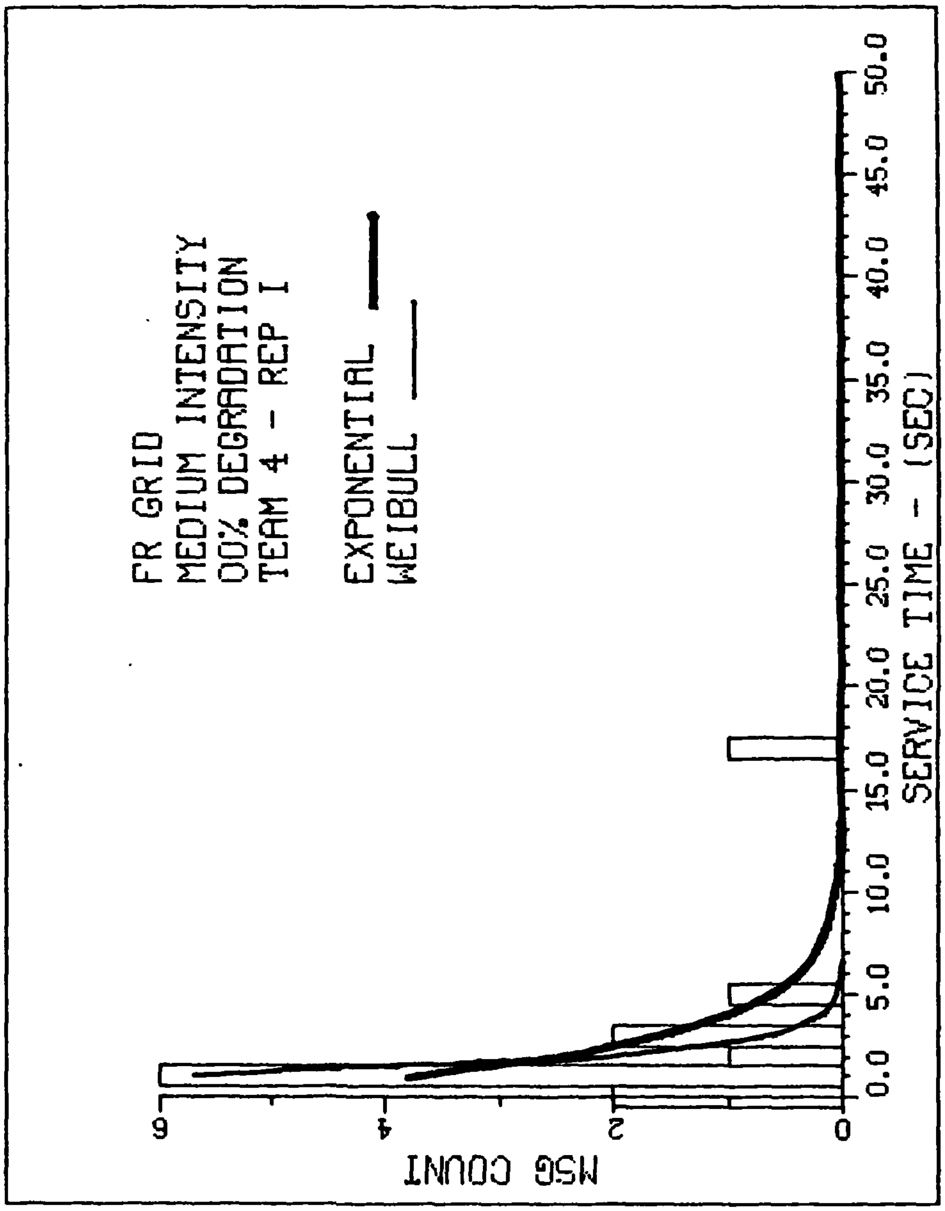
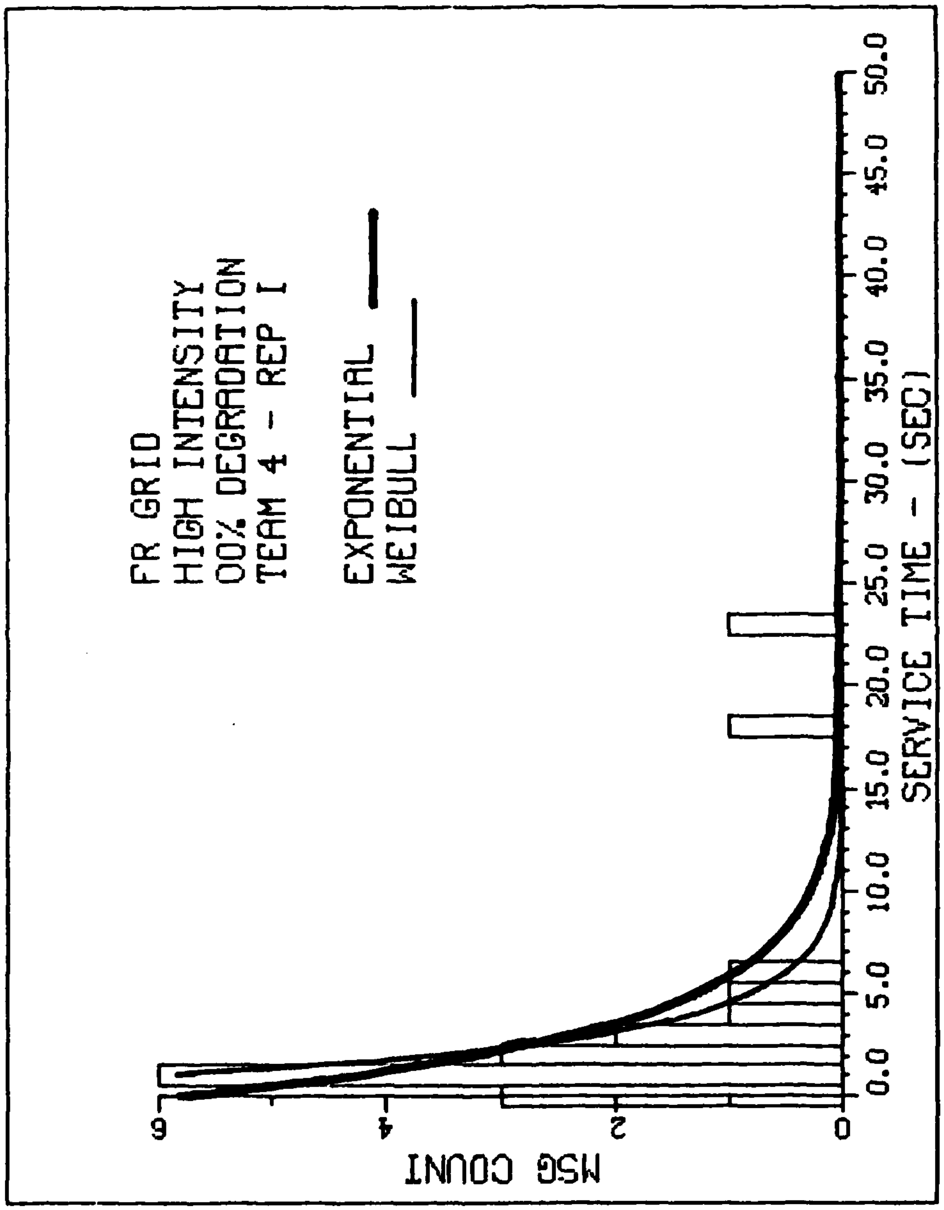
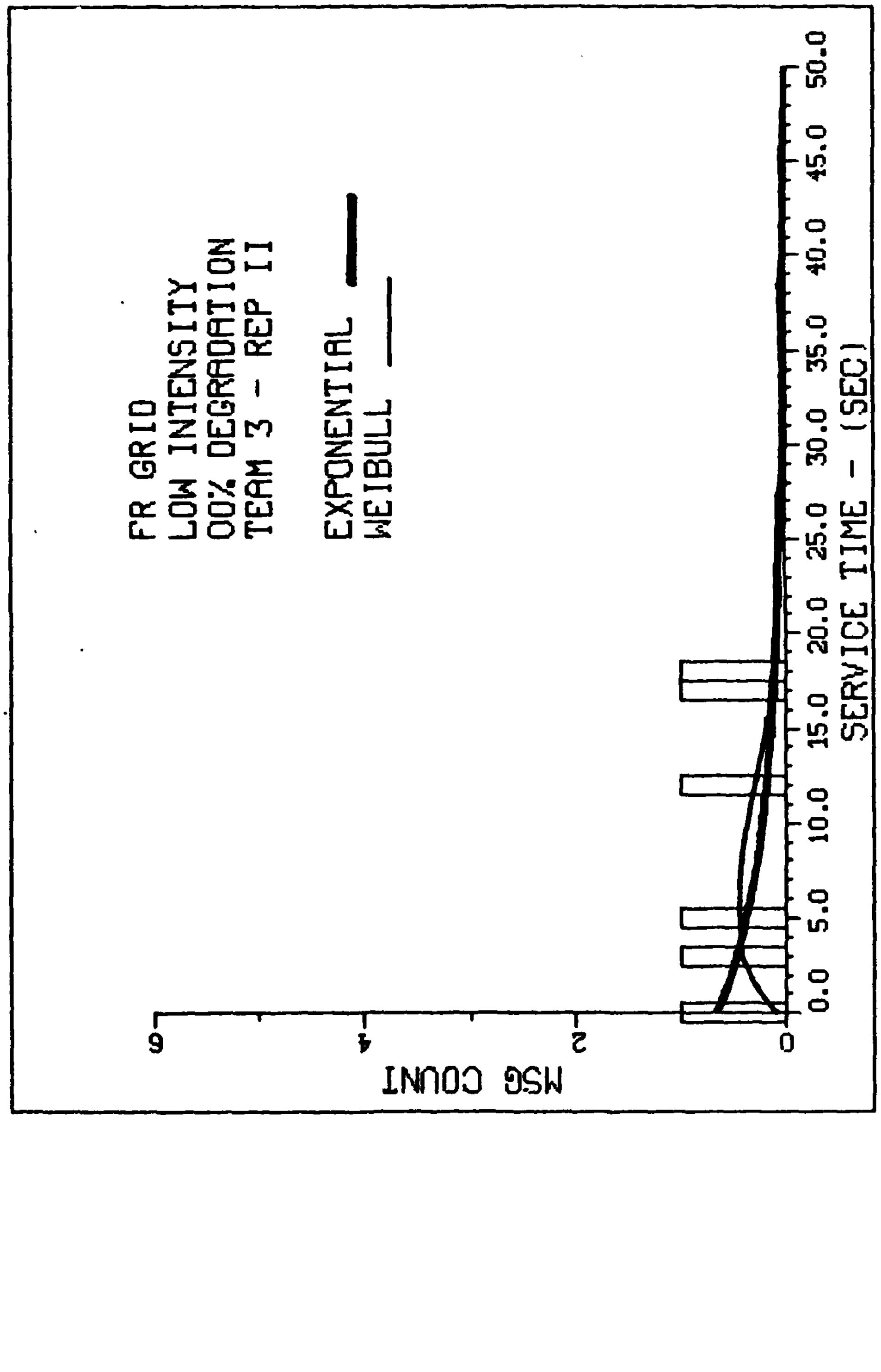
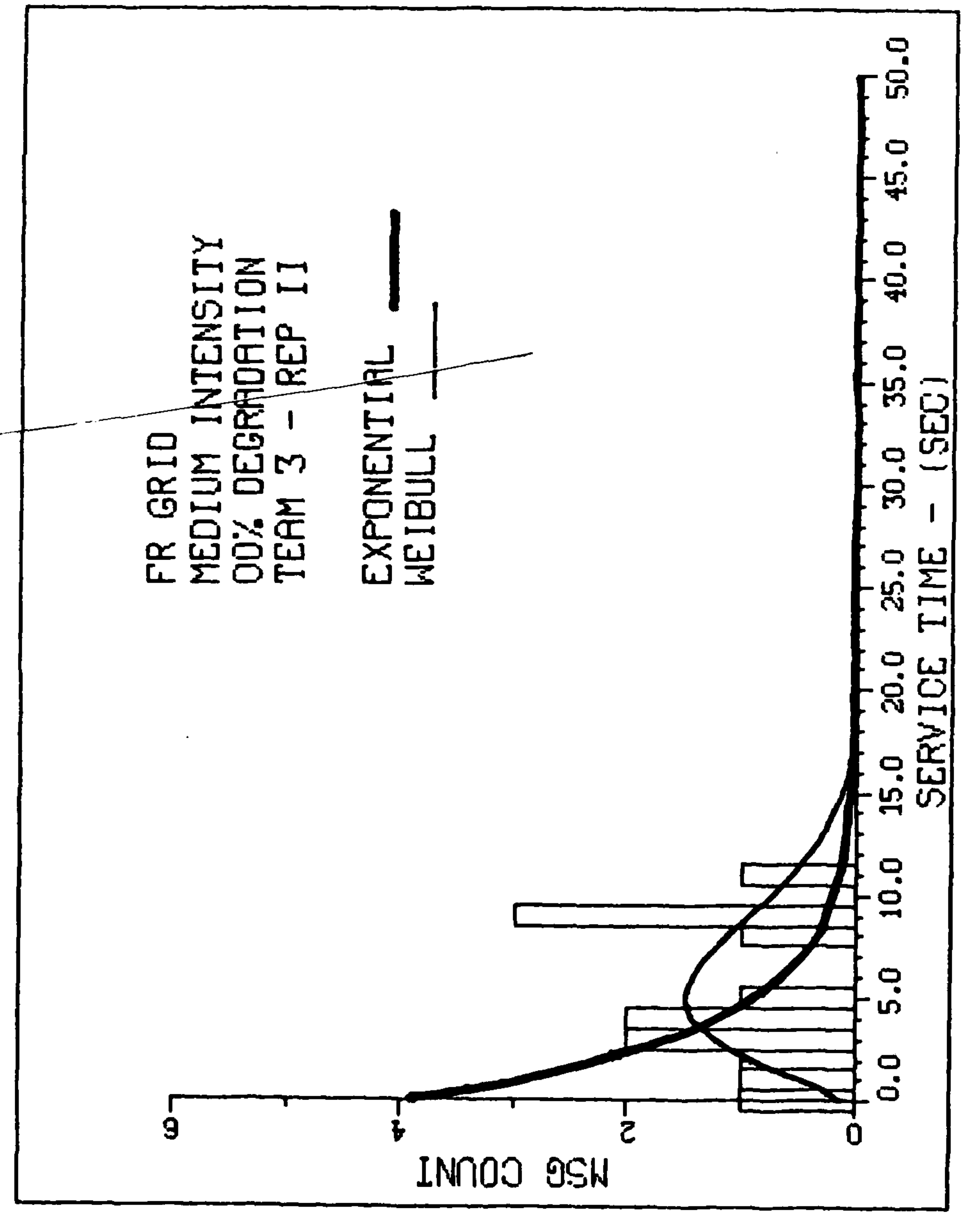
LOW INTENSITY
00% DEGRADATION
TEAM 4 - REP I
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
SERVICE TIME - (SEC)



FR GRID
MEDIUM INTENSITY
00% DEGRADATION
TEAM 4 - REP I
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
SERVICE TIME - (SEC)
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0


FR GRID
HIGH INTENSITY
00% DEGRADATION
TEAM 4 - REP I
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
SERVICE TIME - (SEC)


FR GRID
LOW INTENSITY
00% DEGRADATION
TEAM 3 - REP II
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
SERVICE TIME - (SEC)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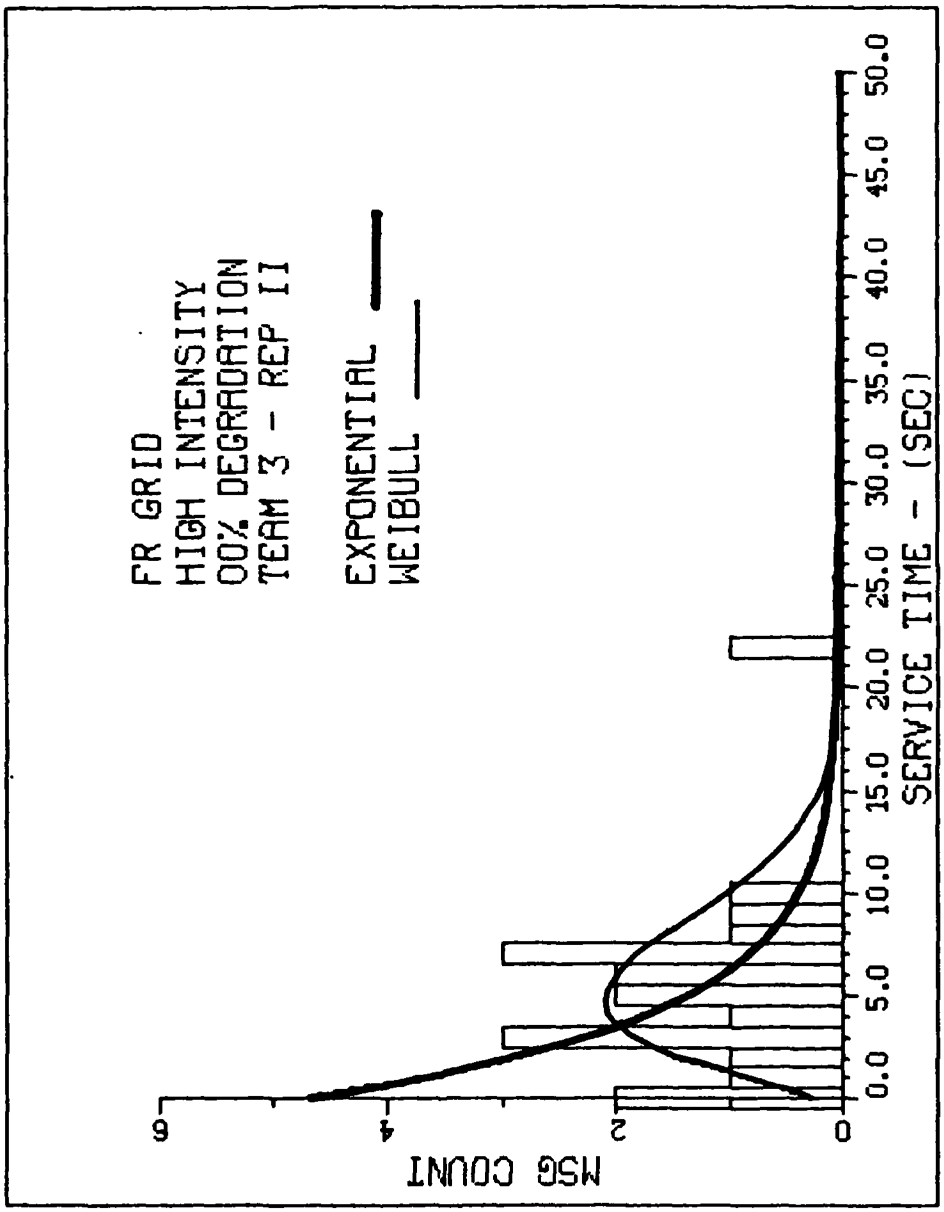
MEDIUM INTENSITY
00% DEGRADATION
TEAM 3 - REP II
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
SERVICE TIME - (SEC)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00% DEGRADATION
TEAM 3 - REP II
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
SERVICE TIME - (SEC)
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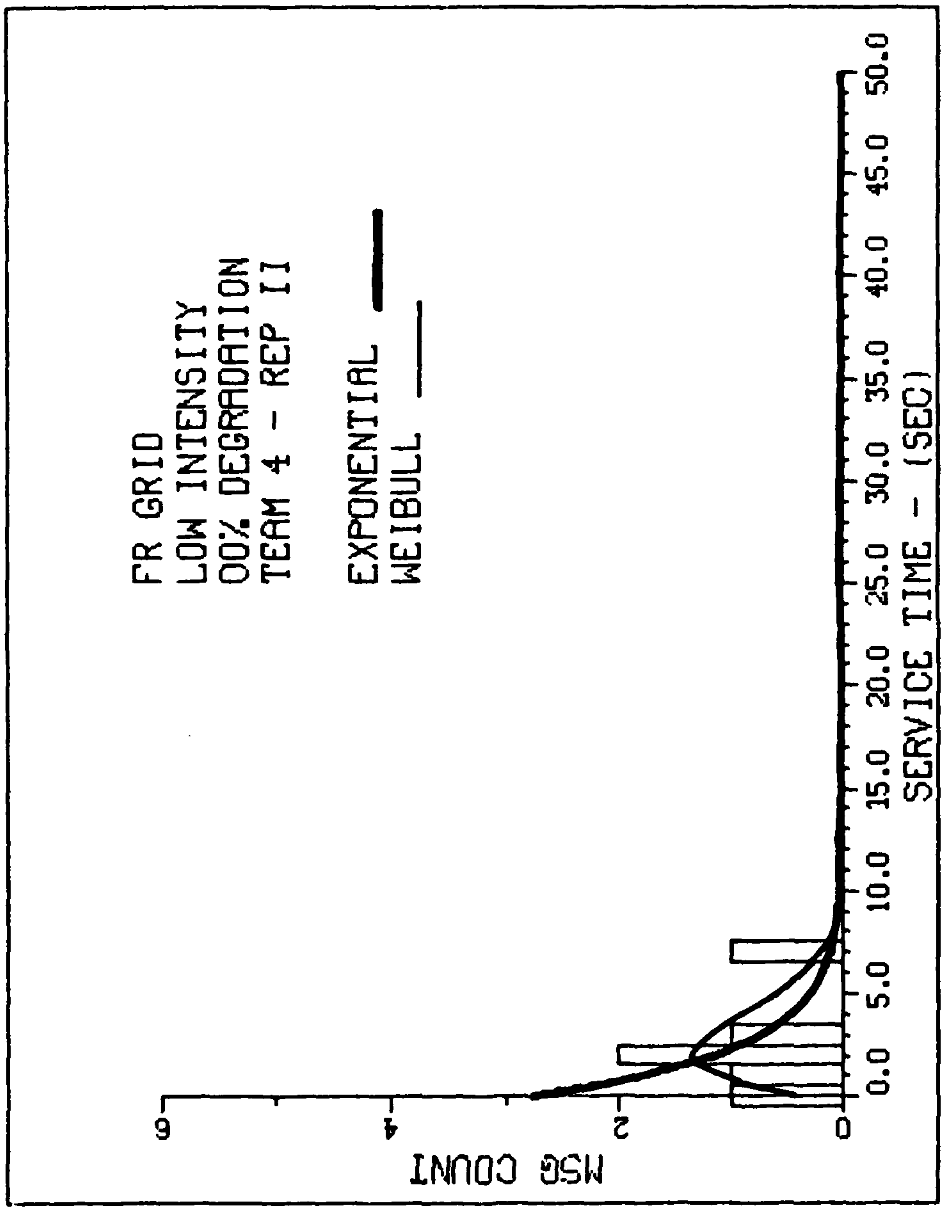
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0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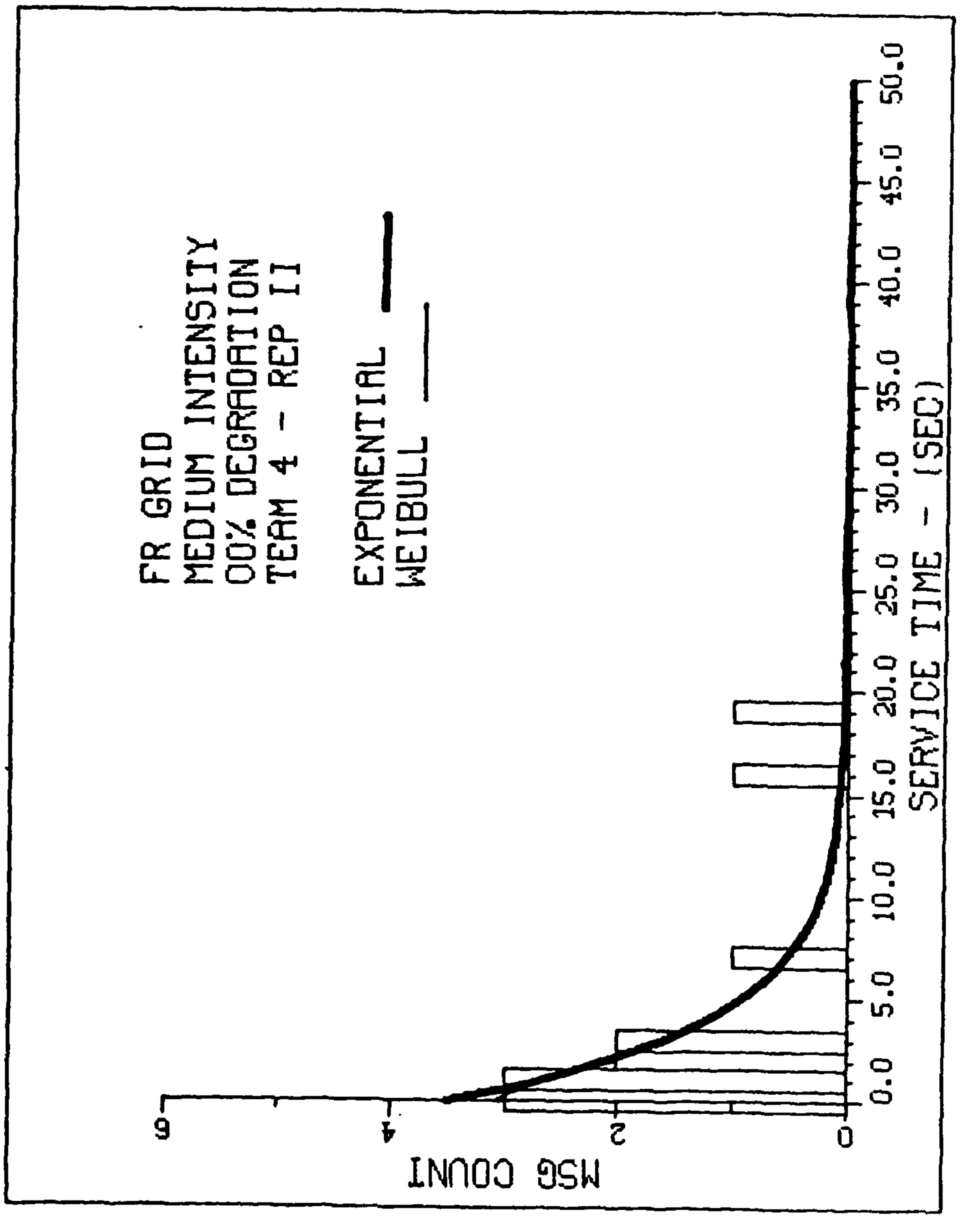
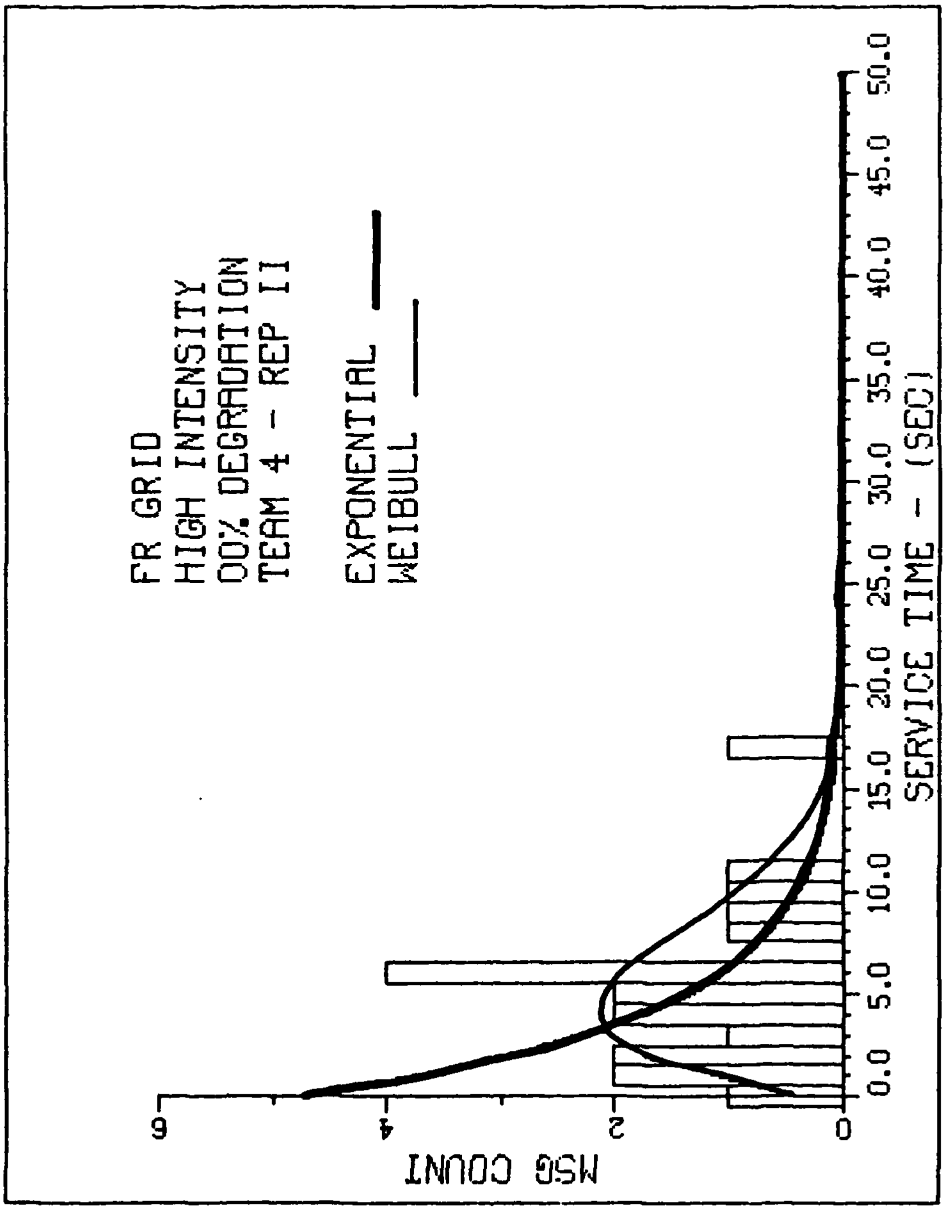
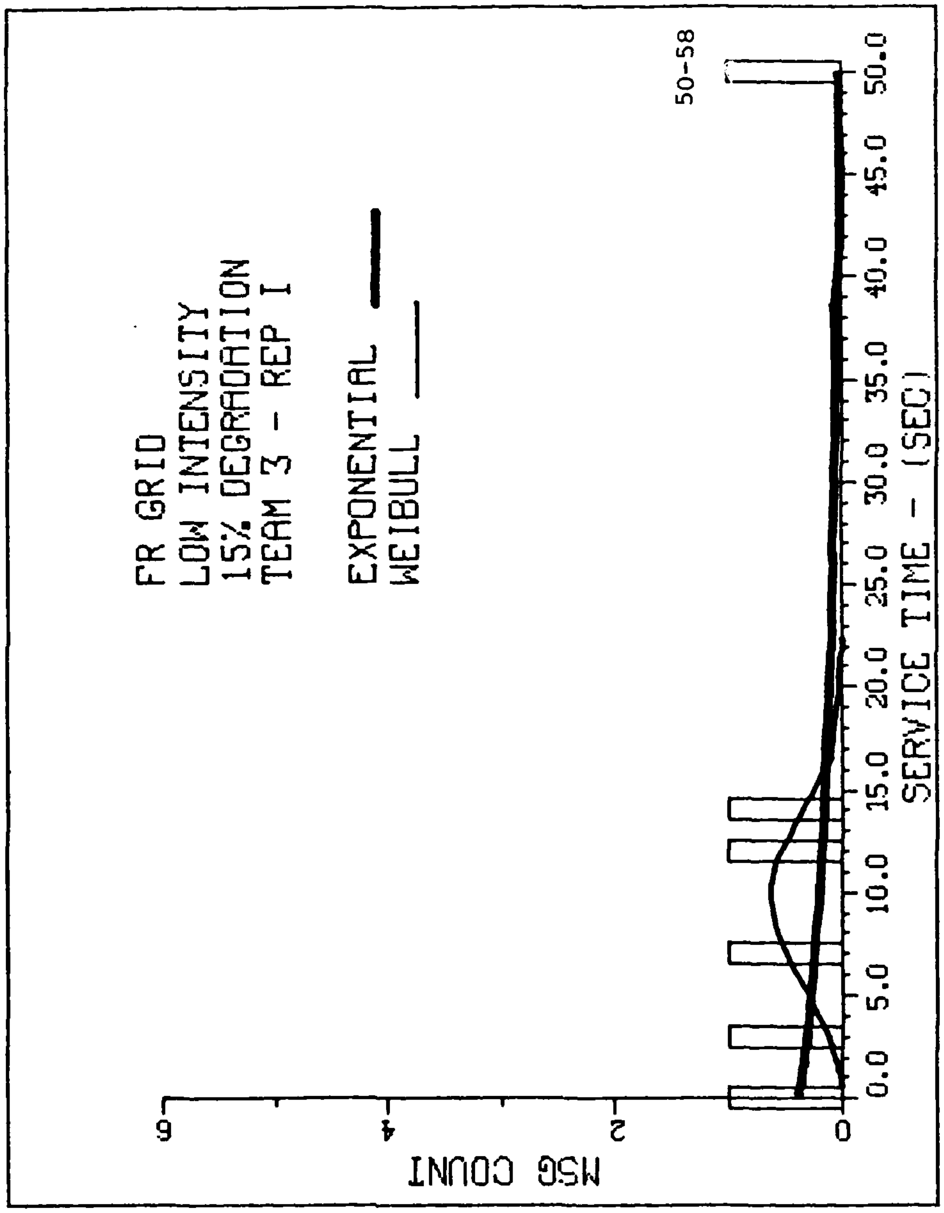
LOW INTENSITY
00% DEGRADATION
TEAM 4 - REP II
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
SERVICE TIME - (SEC)


FR GRID
MEDIUM INTENSITY
00% DEGRADATION
TEAM 4 - REP II
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
SERVICE TIME - (SEC)


FR GRID
HIGH INTENSITY
00% DEGRADATION
TEAM 4 - REP II
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
SERVICE TIME - (SEC)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LOW INTENSITY
15% DEGRADATION
TEAM 3 - REP I
EXPONENTIAL
WEIBULL
50-58
MSG COUNT
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4
6
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15.0
20.0
25.0
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40.0
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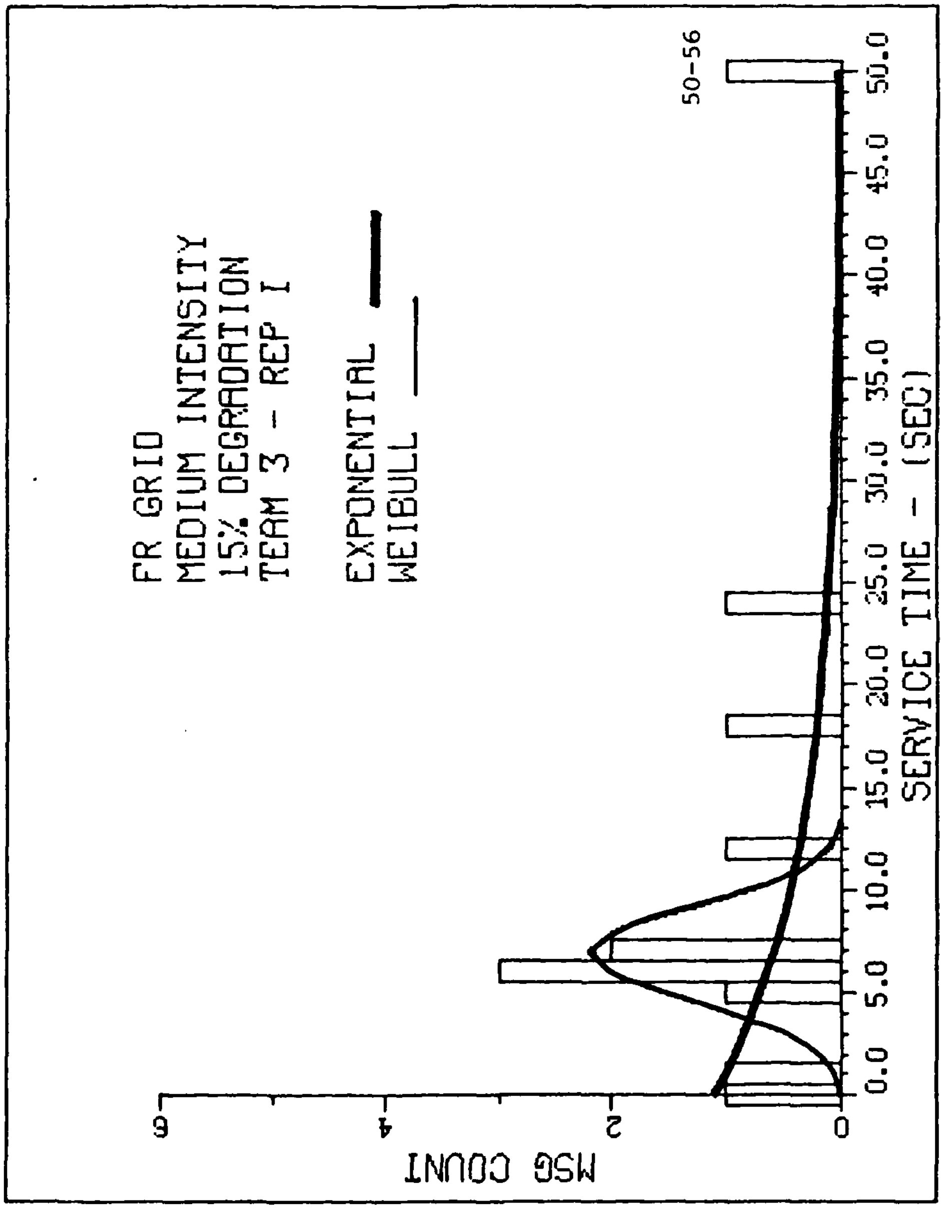
50.0
SERVICE TIME - (SEC)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MEDIUM INTENSITY
15% DEGRADATION
TEAM 3 - REP I
EXPONENTIAL
WEIBULL
50-56
MSG COUNT
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4
6
SERVICE TIME - (SEC)
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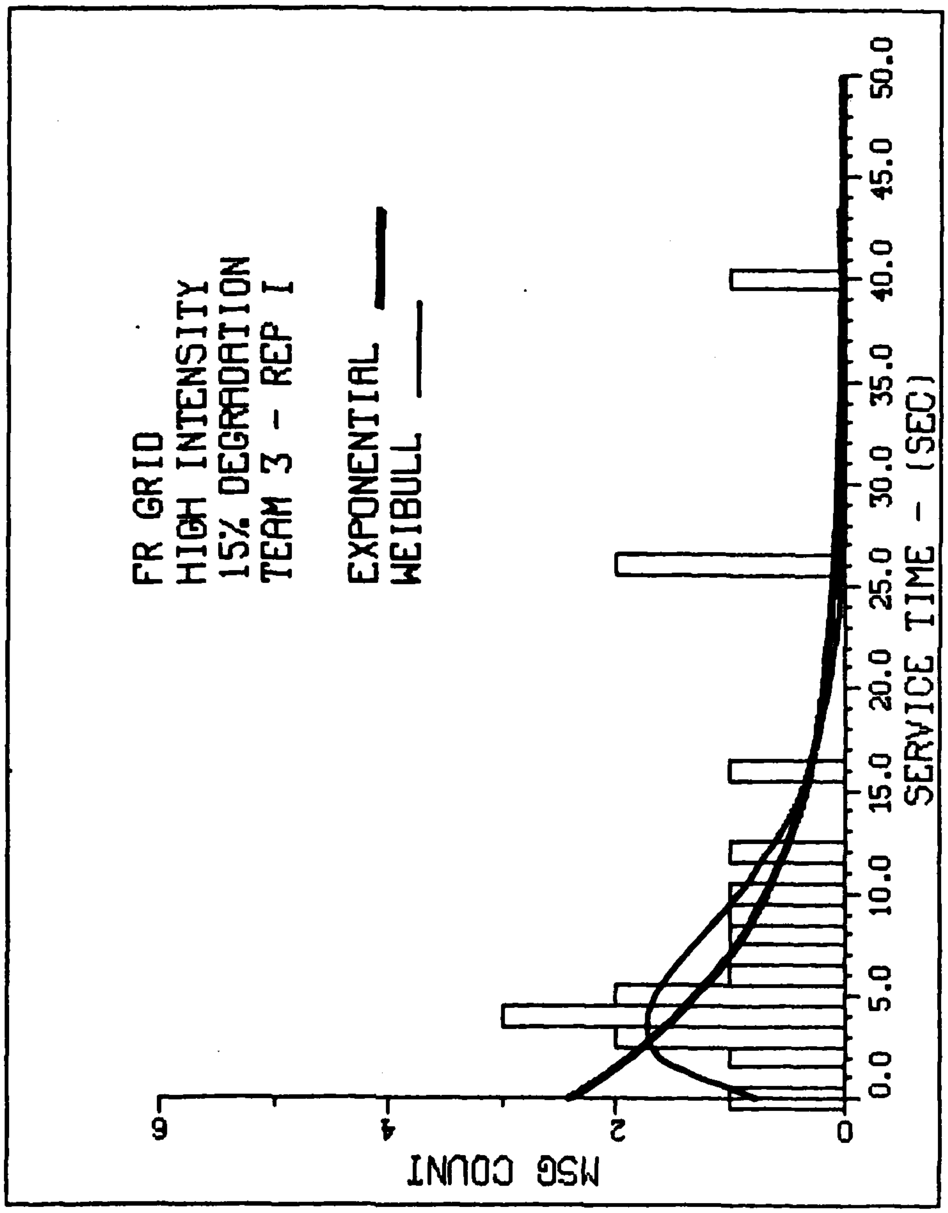
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15% DEGRADATION
TEAM 3 - REP I
EXPONENTIAL
WEIBULL
MSG COUNT
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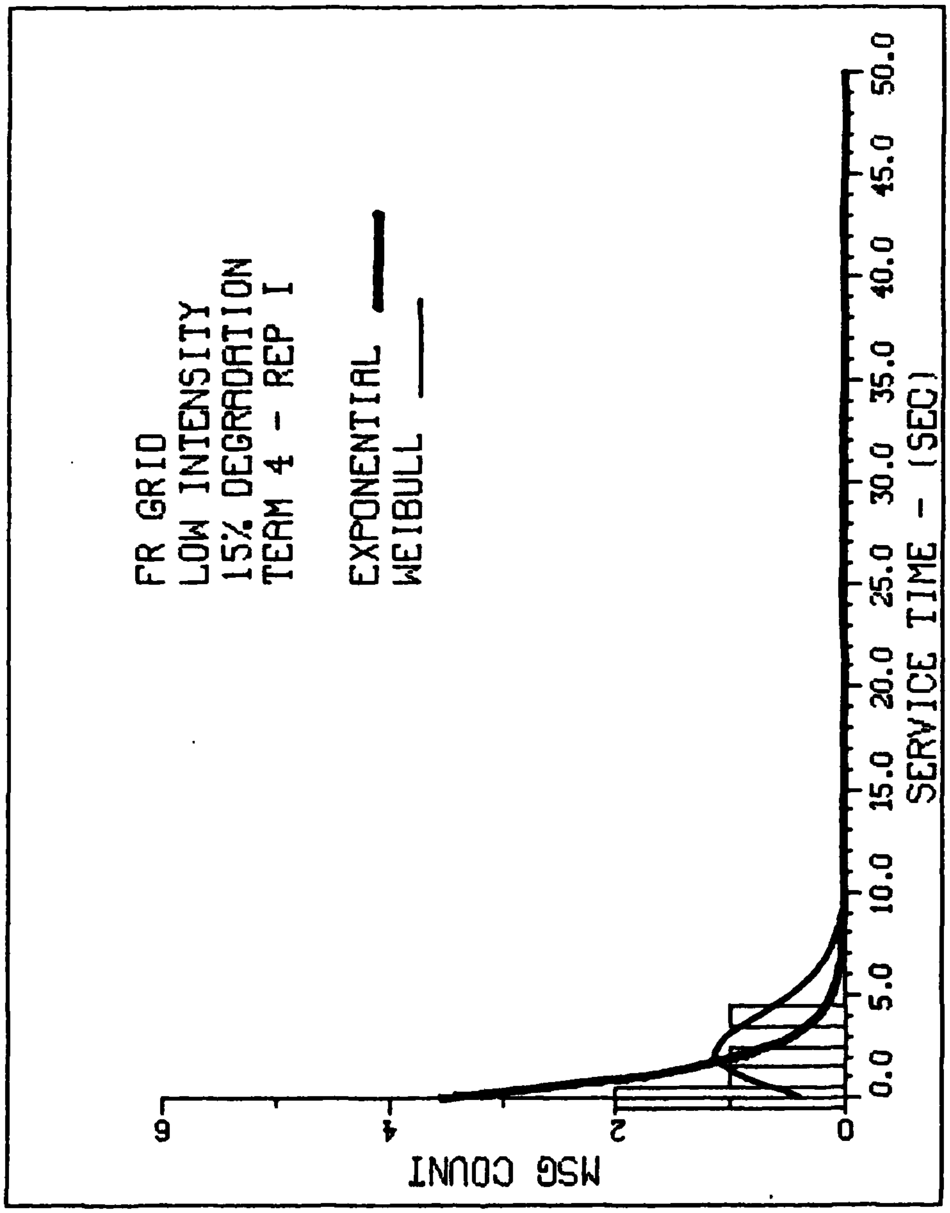
40.0
45.0
50.0
SERVICE TIME - (SEC)



FR GRID
LOW INTENSITY
15% DEGRADATION
TEAM 4 - REP I
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
SERVICE TIME - (SEC)
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10.0
15.0
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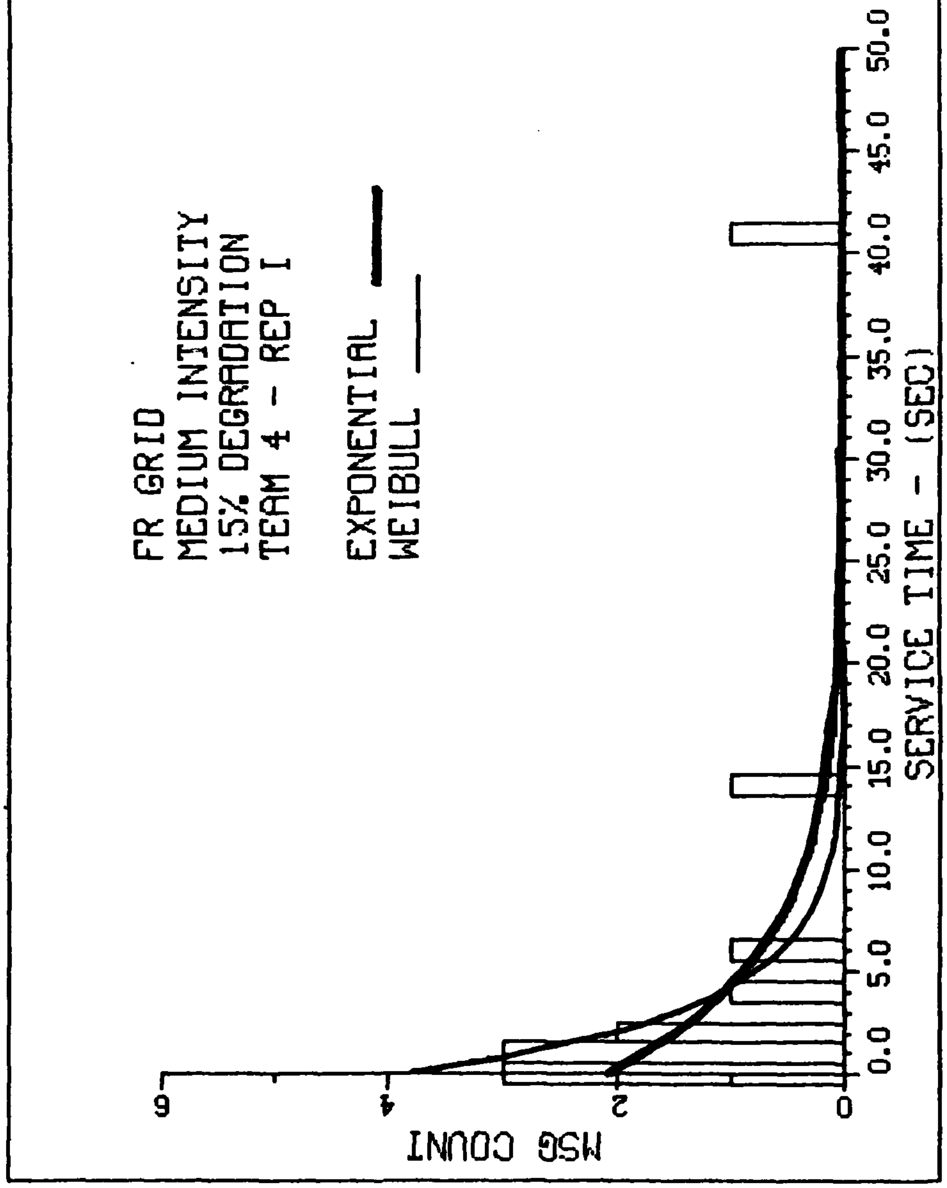
FR GRID
MEDIUM INTENSITY
15% DEGRADATION
TEAM 4 - REP I
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
SERVICE TIME - (SEC)
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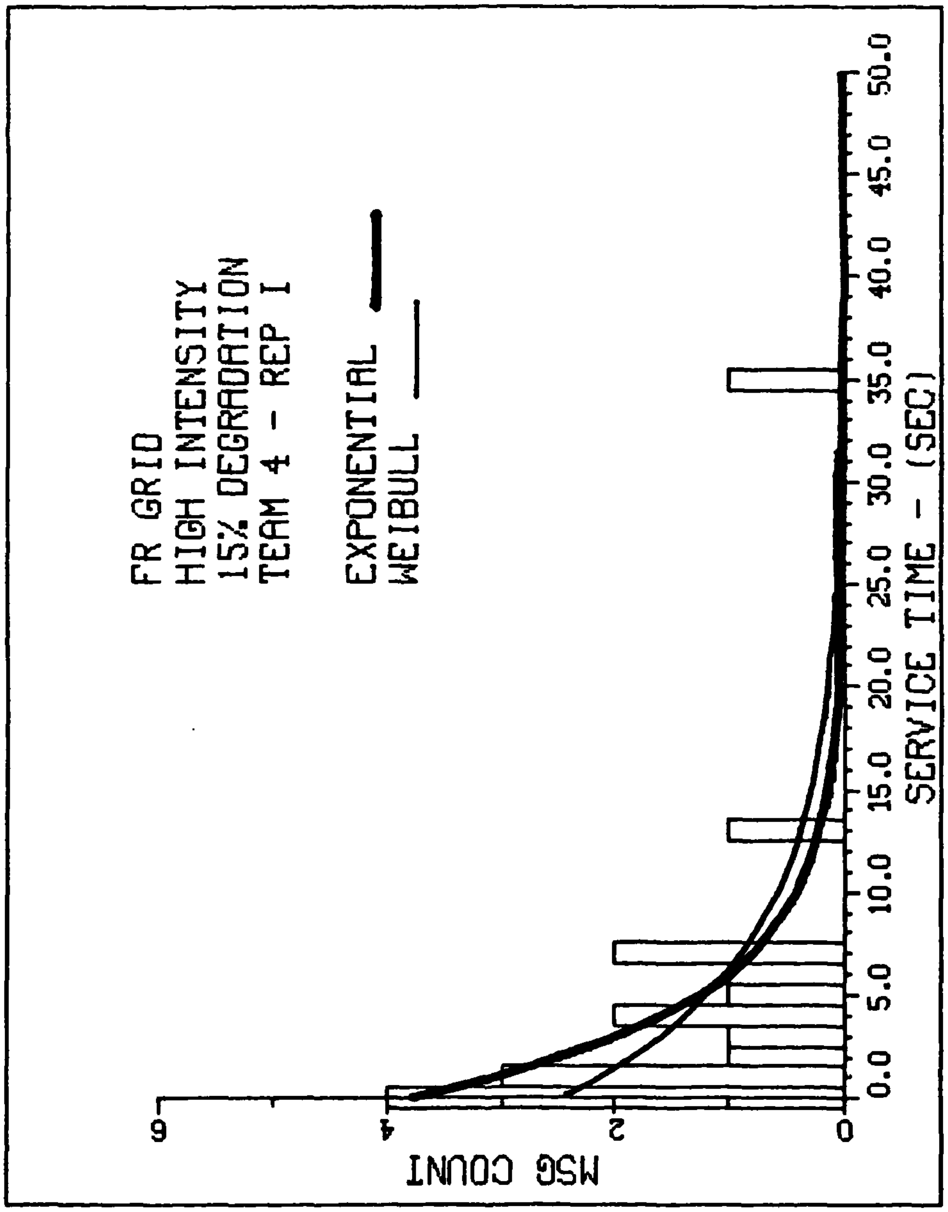
FR GRID
HIGH INTENSITY
15% DEGRADATION
TEAM 4 - REP I
EXPONENTIAL
WEIBULL
MSG COUNT
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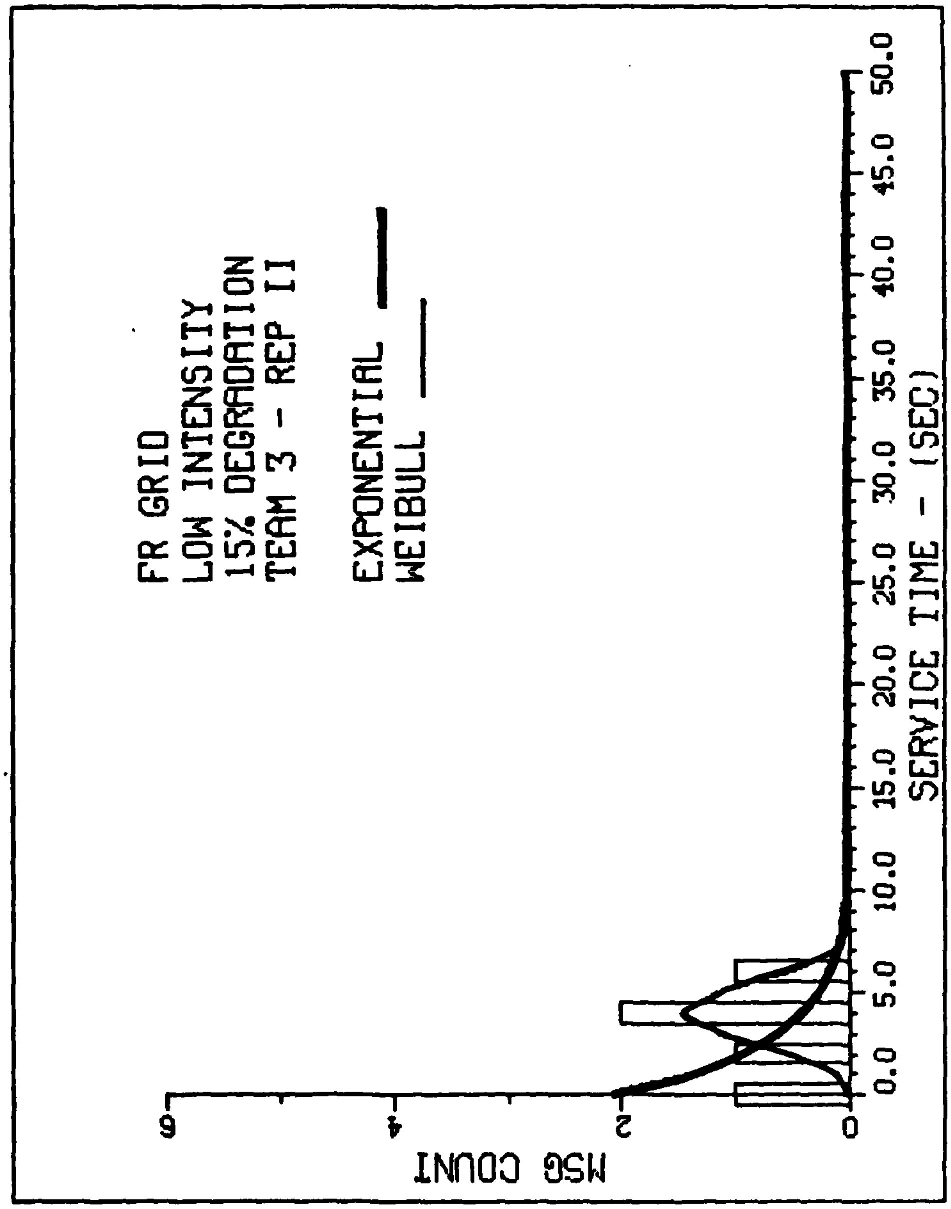
40.0
45.0
50.0
SERVICE TIME - (SEC)


FR GRID
LOW INTENSITY
15% DEGRADATION
TEAM 3 - REP II
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
SERVICE TIME - (SEC)
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10.0
15.0
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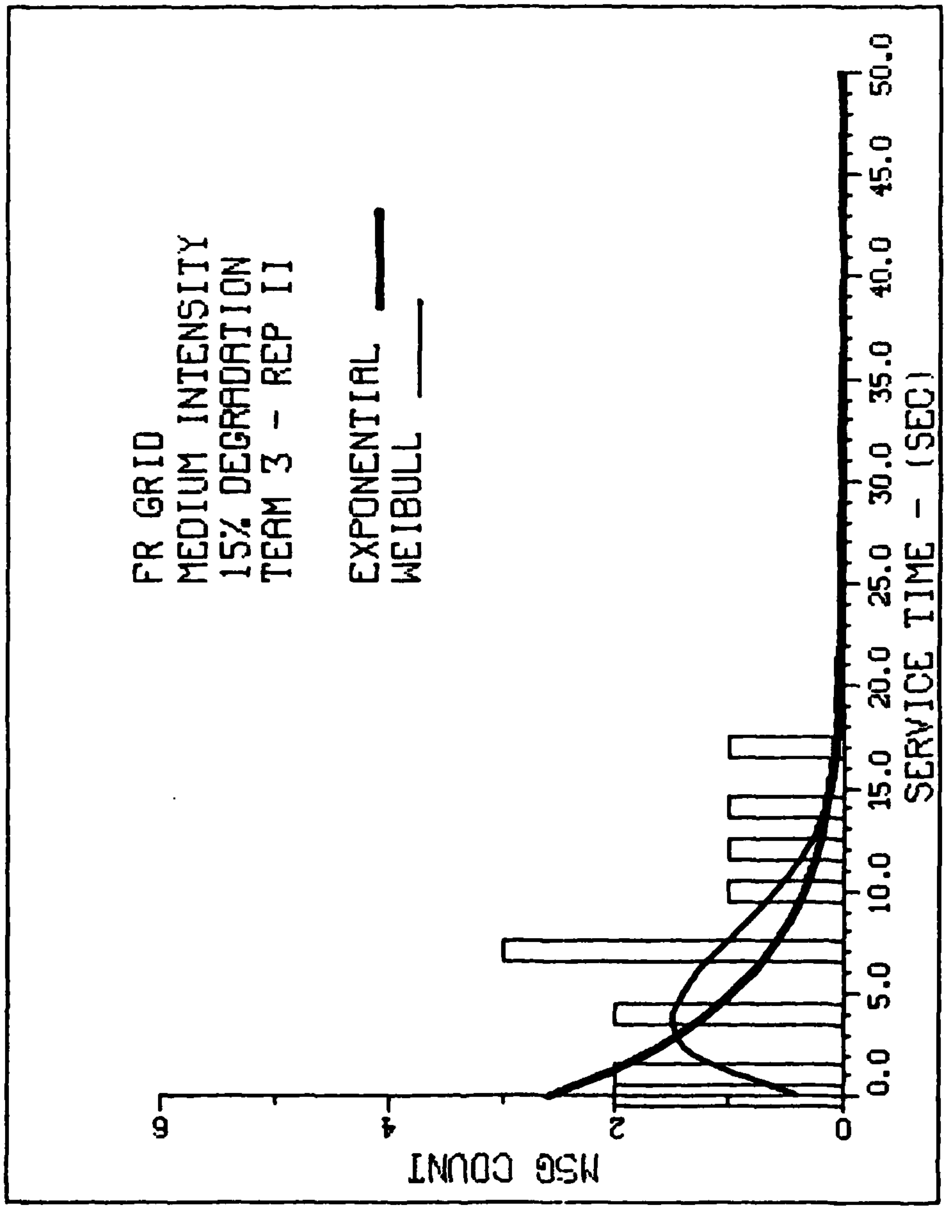
50.0


FR GRID
MEDIUM INTENSITY
15% DEGRADATION
TEAM 3 - REP II
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
0.0
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20.0
25.0
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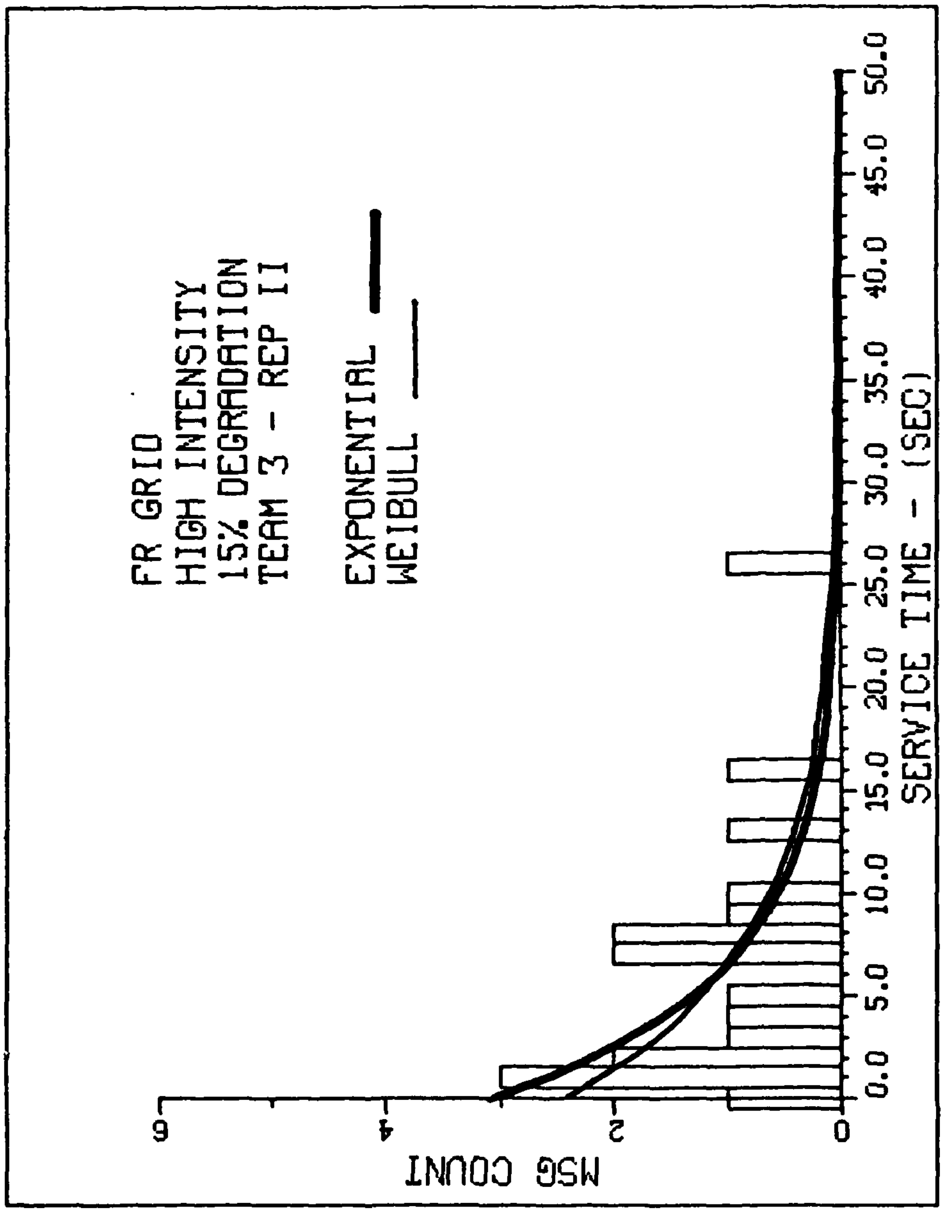
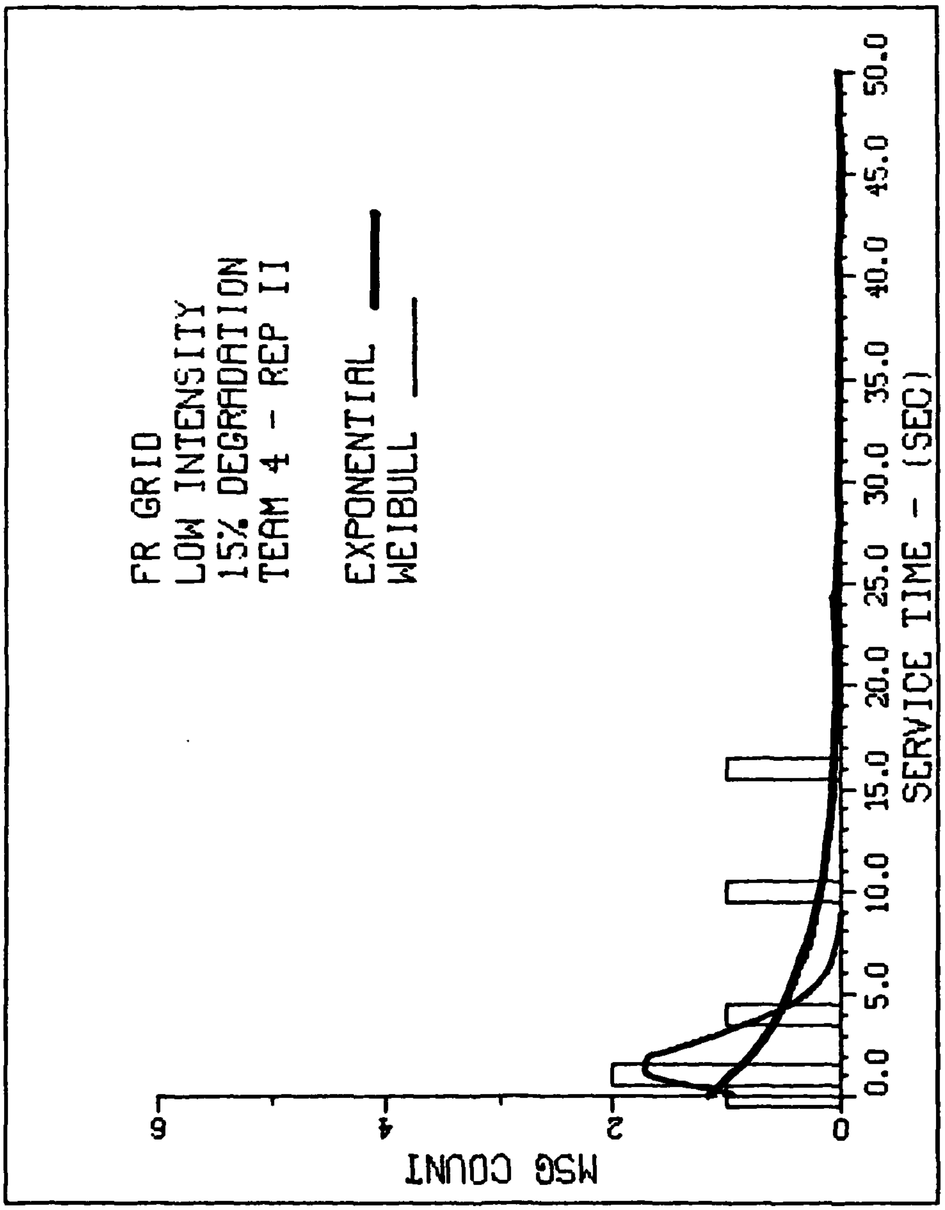
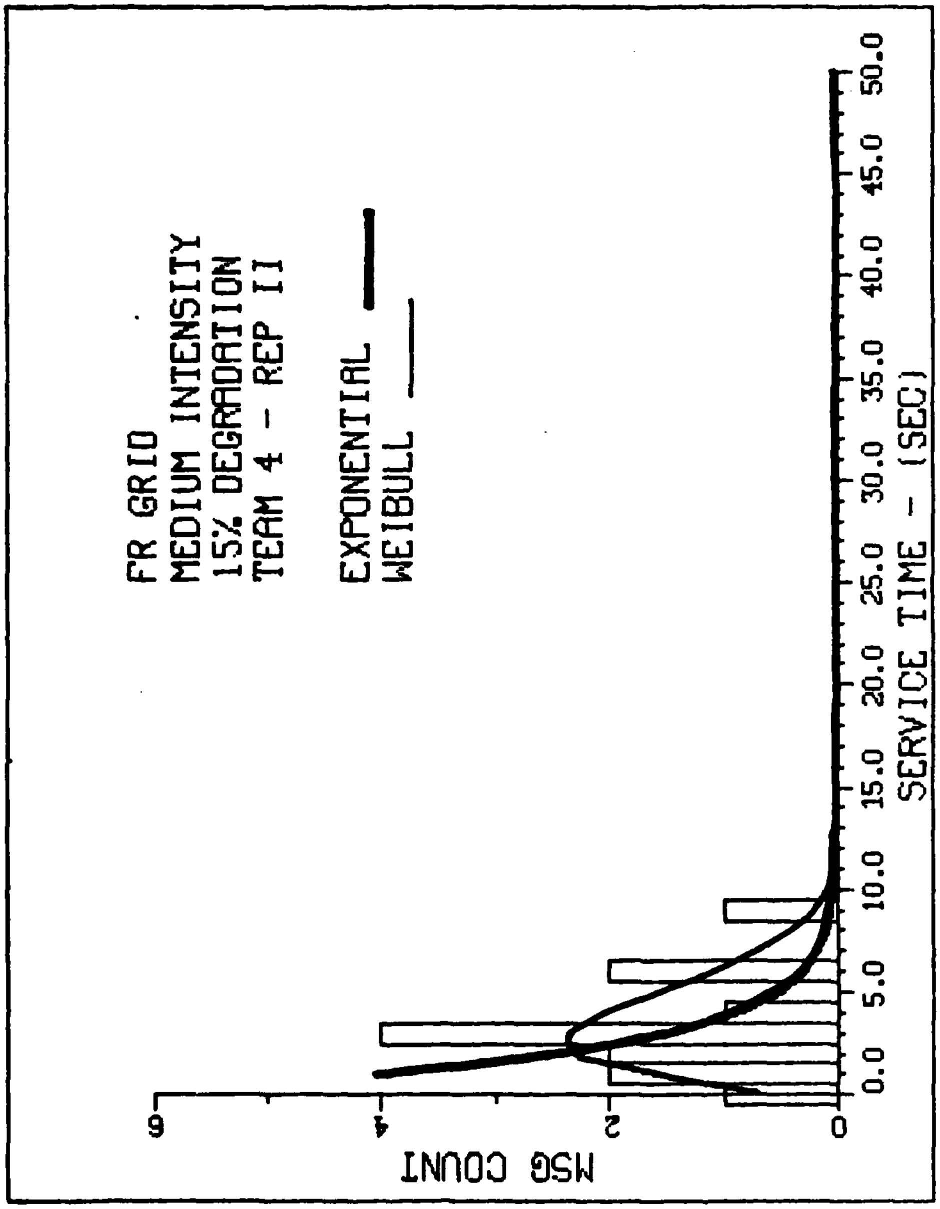
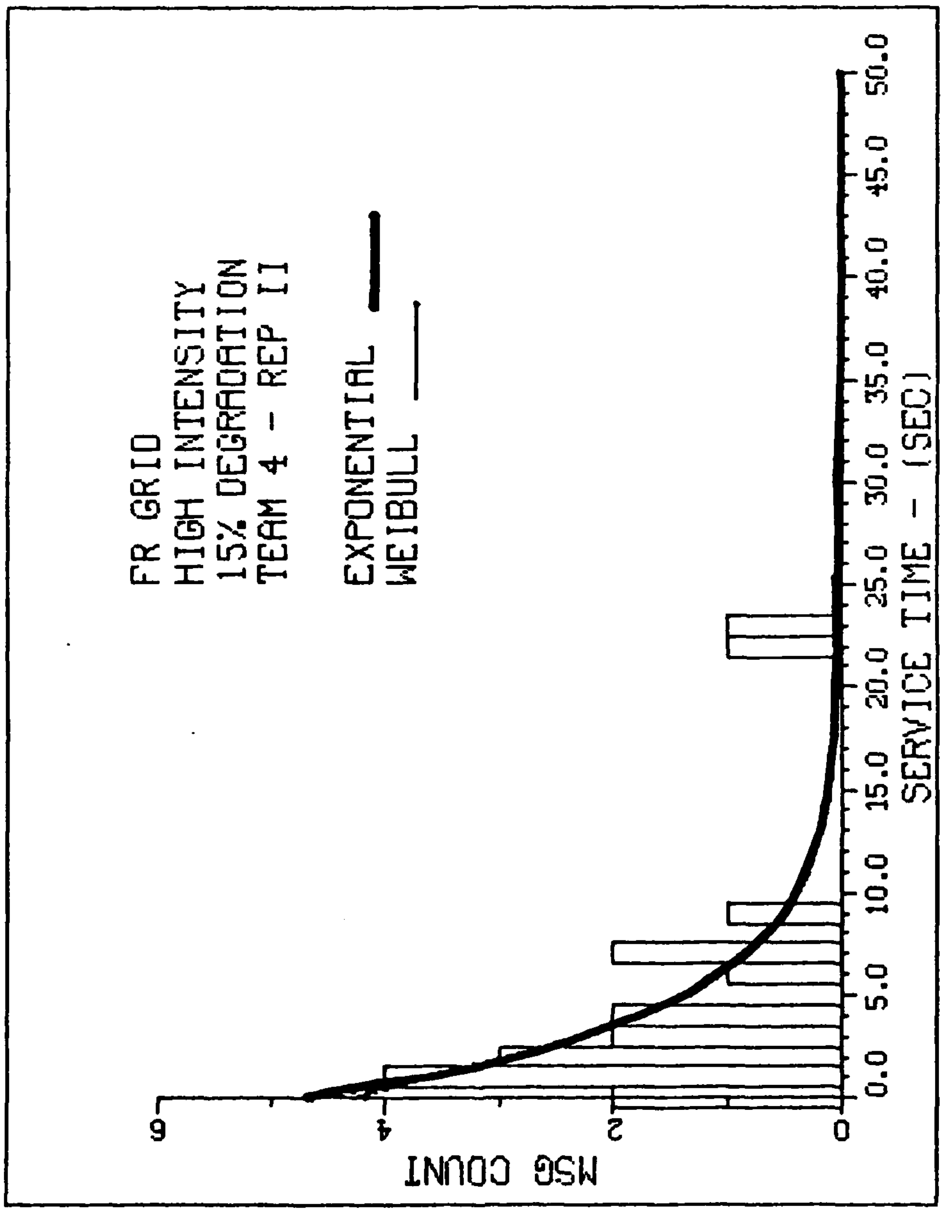
40.0
45.0
50.0
SERVICE TIME - (SEC)


FR GRID
HIGH INTENSITY
15% DEGRADATION
TEAM 3 - REP II
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
SERVICE TIME - (SEC)


FR GRID
LOW INTENSITY
15% DEGRADATION
TEAM 4 - REP II
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
SERVICE TIME - (SEC)


FR GRID
MEDIUM INTENSITY
15% DEGRADATION
TEAM 4 - REP II
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
SERVICE TIME - (SEC)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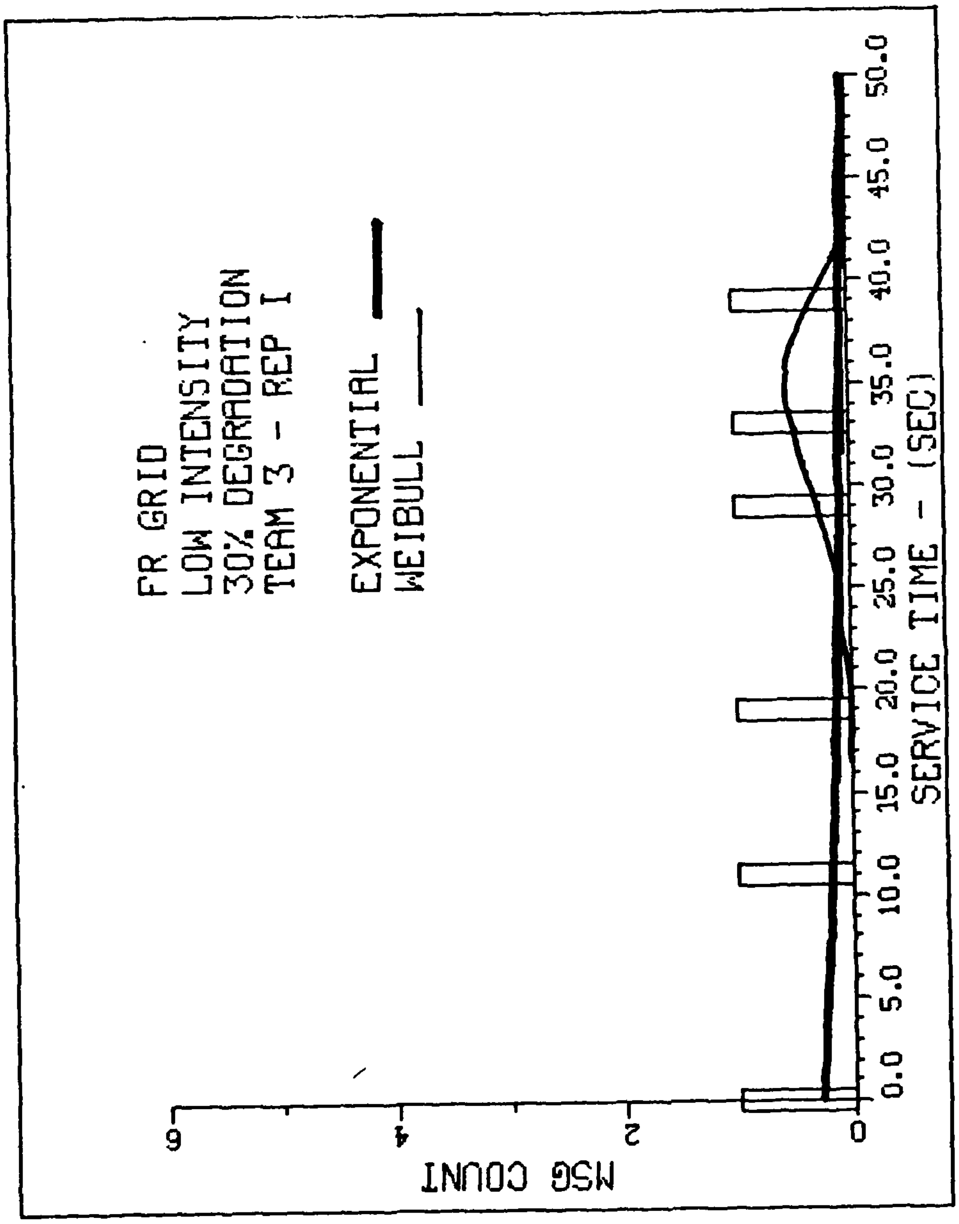
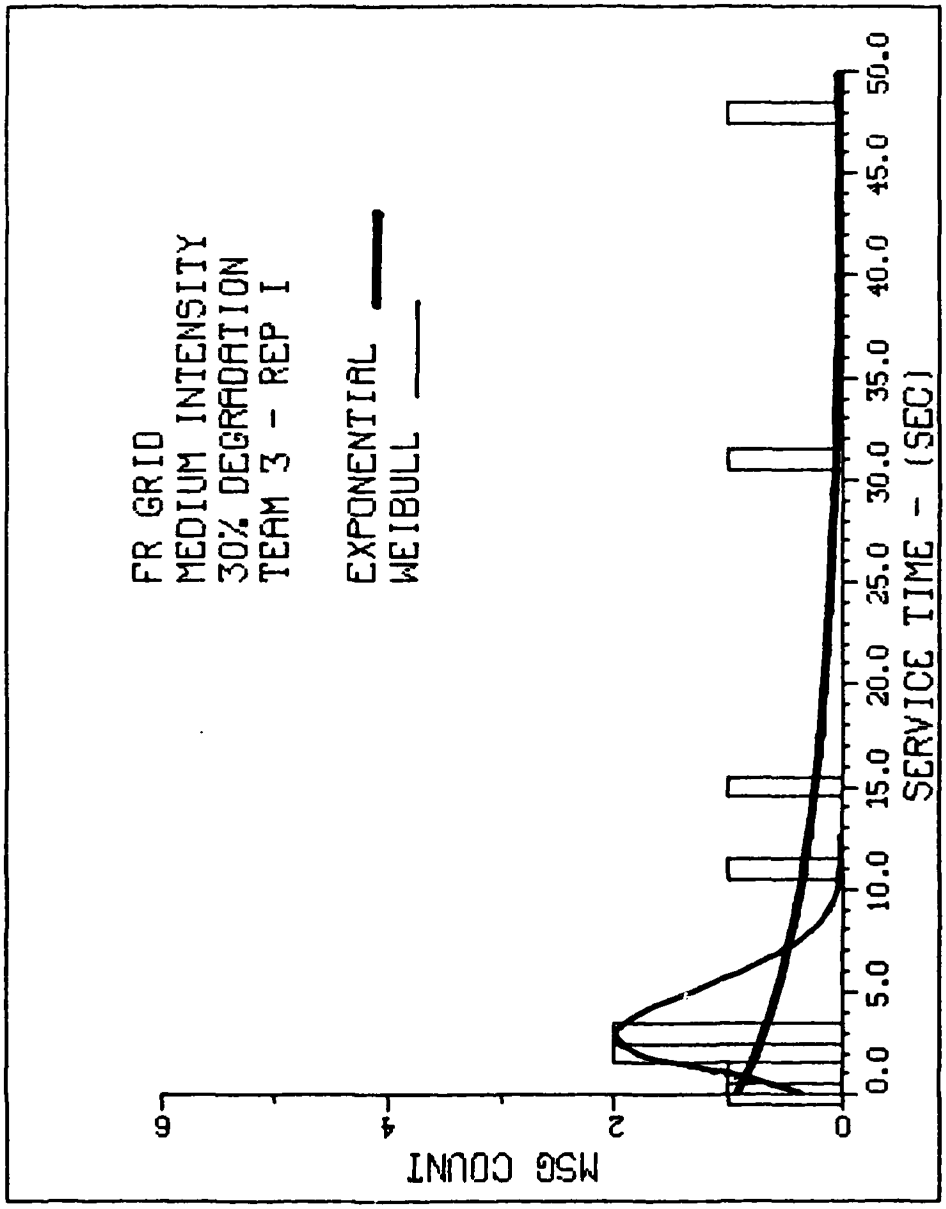
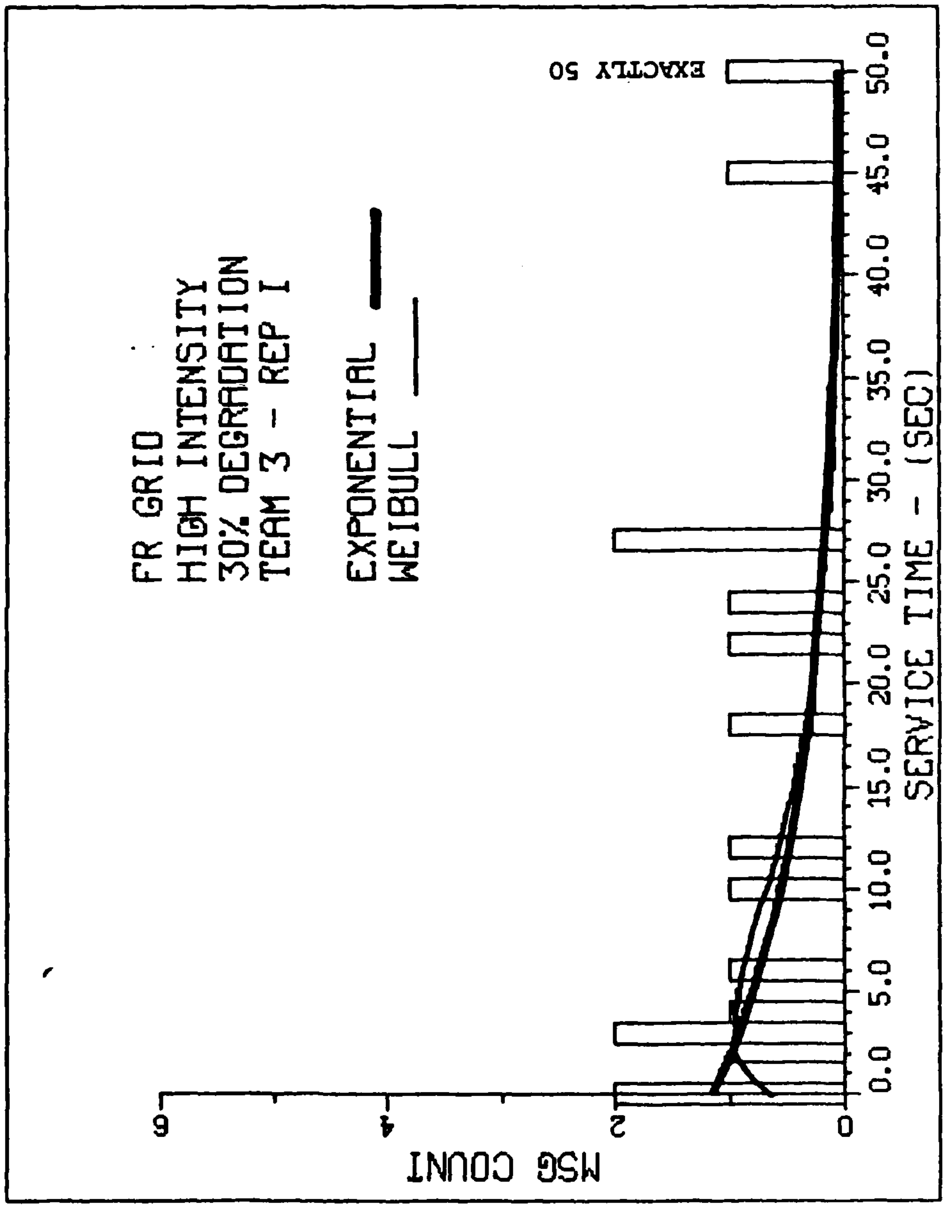
HIGH INTENSITY
15% DEGRADATION
TEAM 4 - REP II
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
SERVICE TIME - (SEC)


FR GRID
LOW INTENSITY
30% DEGRADATION
TEAM 3 - REP I
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
SERVICE TIME - (SEC)


FR GRID
MEDIUM INTENSITY
30% DEGRADATION
TEAM 3 - REP I
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
SERVICE TIME - (SEC)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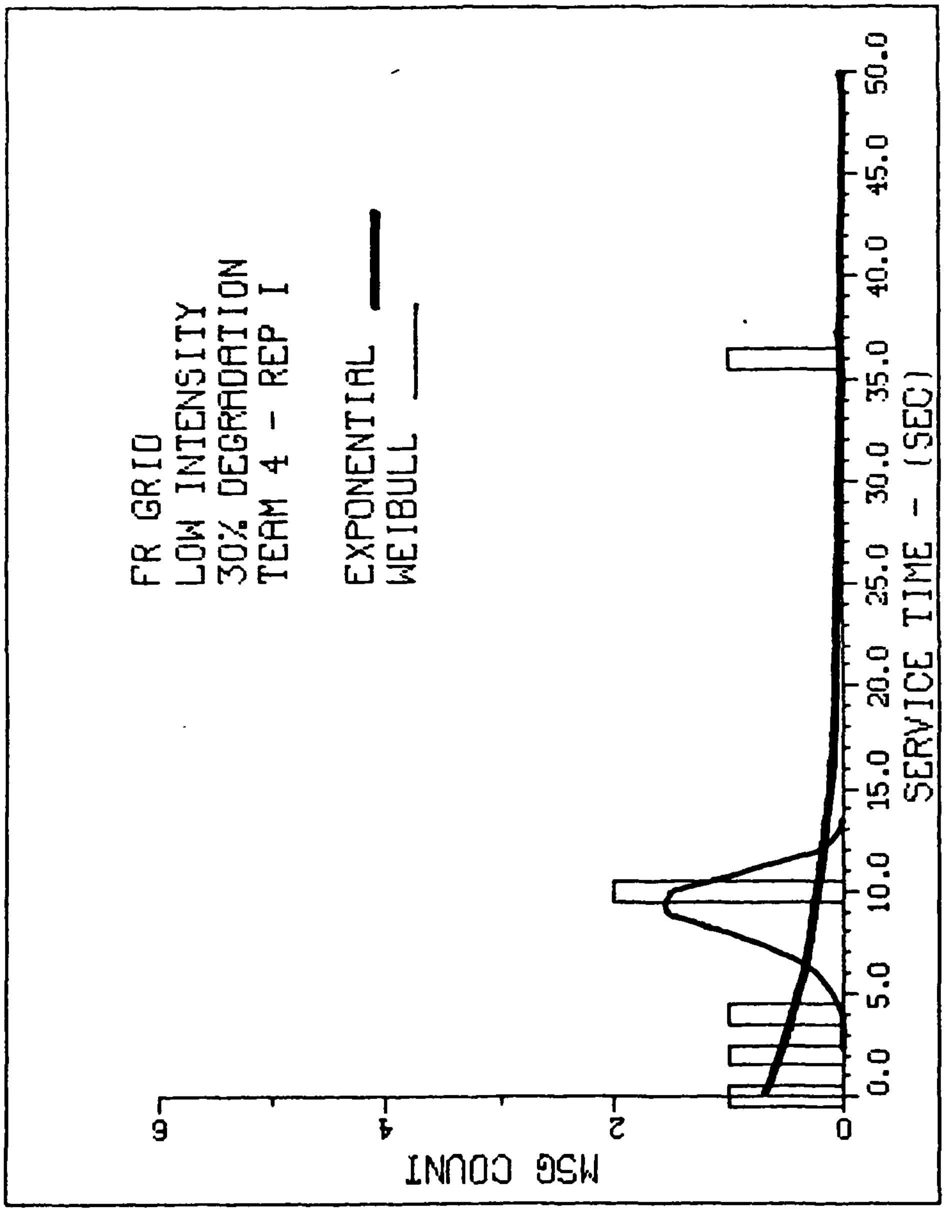
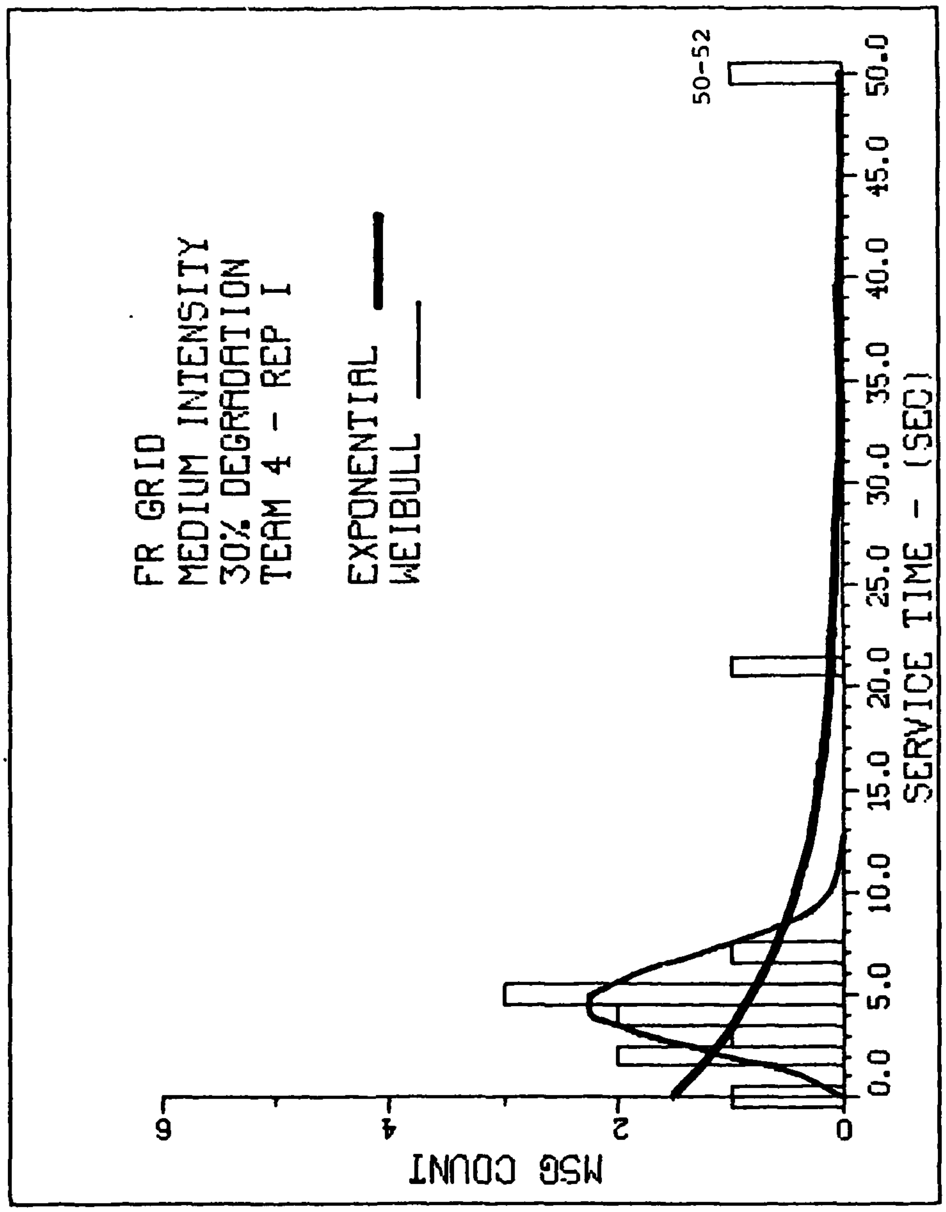
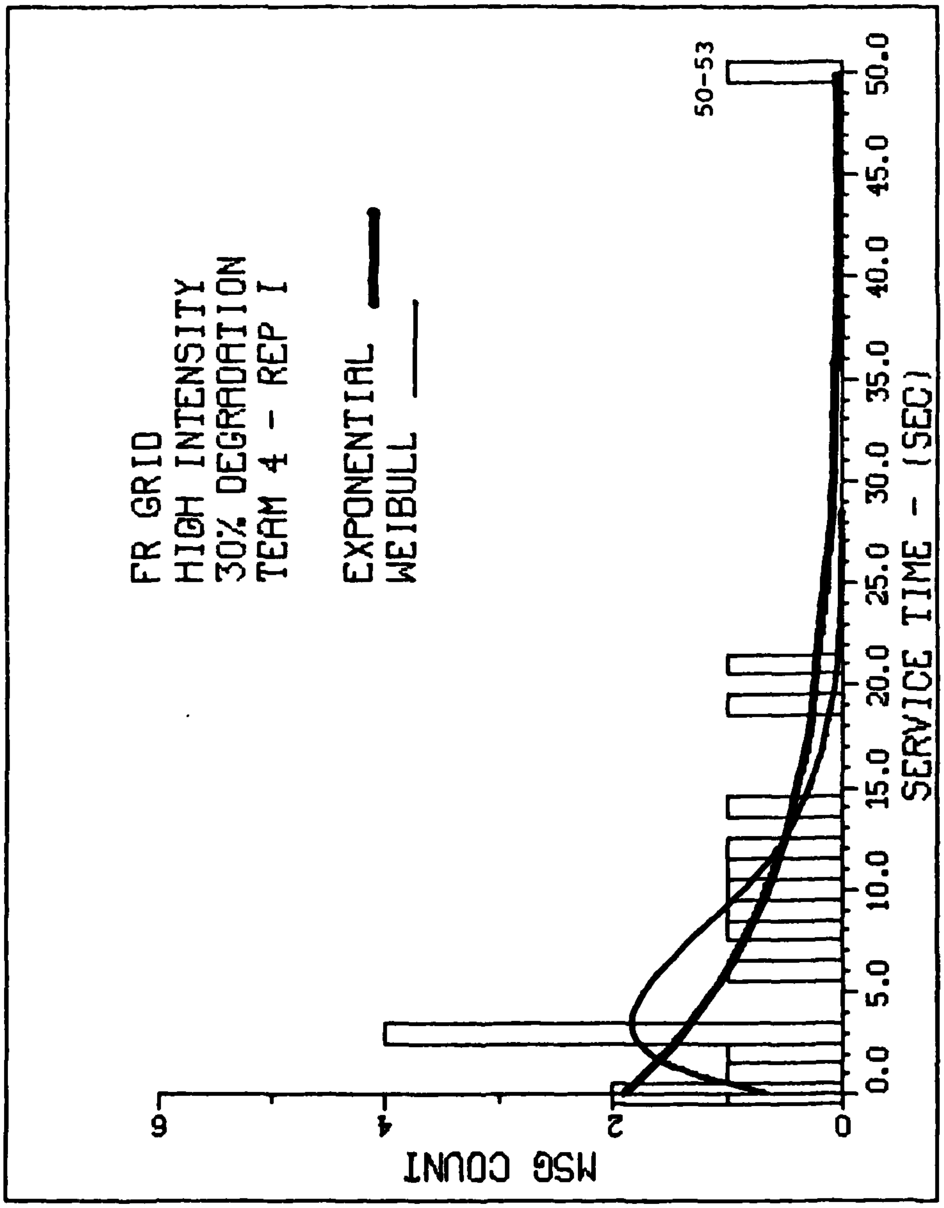
HIGH INTENSITY
30% DEGRADATION
TEAM 3 - REP I
EXPONENTIAL
WEIBULL
EXACTLY 50
MSG COUNT
0
2
4
6
SERVICE TIME - (SEC)
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0


FR GRID
LOW INTENSITY
30% DEGRADATION
TEAM 4 - REP I
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
SERVICE TIME - (SEC)
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0



FR GRID
MEDIUM INTENSITY
30% DEGRADATION
TEAM 4 - REP I
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
50-52
SERVICE TIME - (SEC)



FR GRID
HIGH INTENSITY
30% DEGRADATION
TEAM 4 - REP I
EXPONENTIAL
WEIBULL
50-53
MSG COUNT
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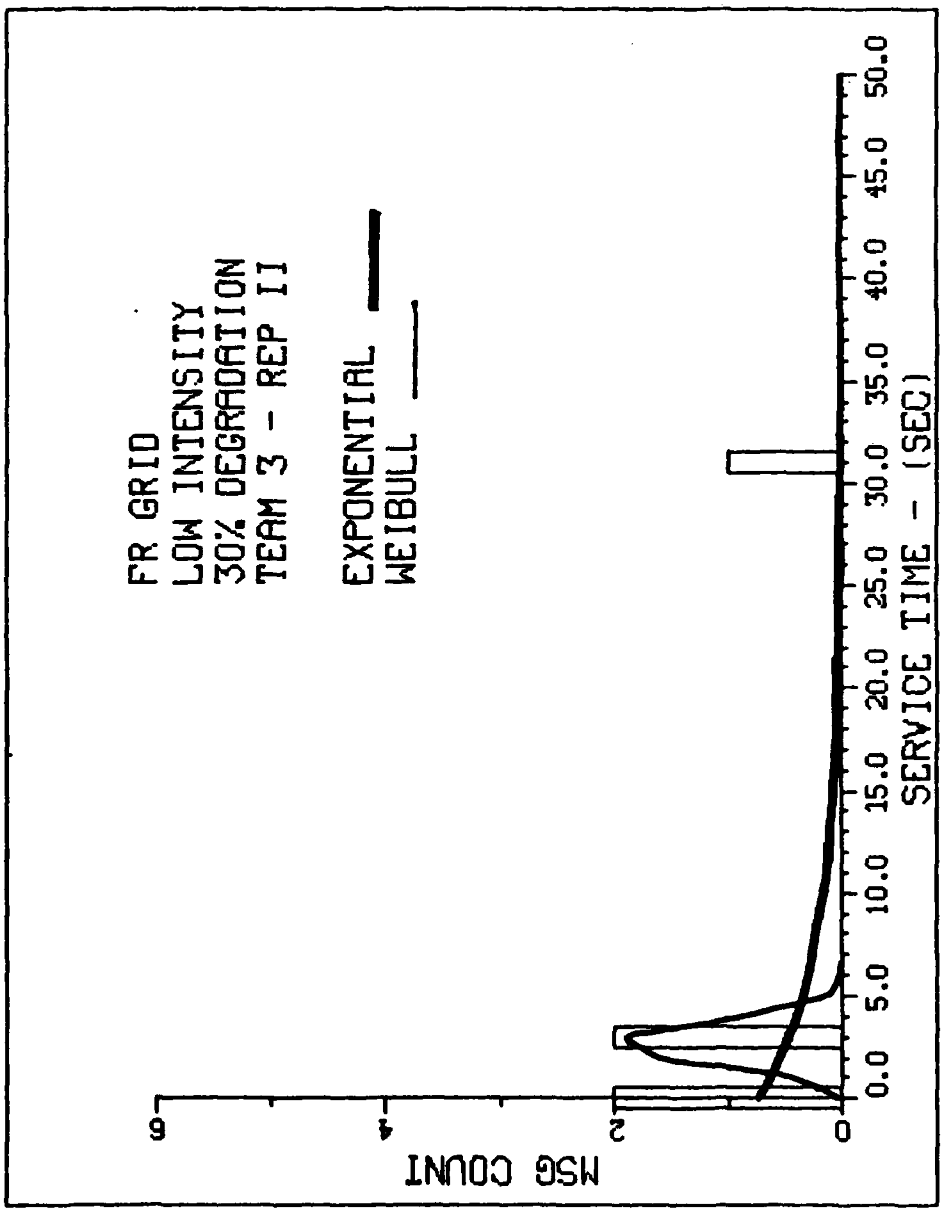
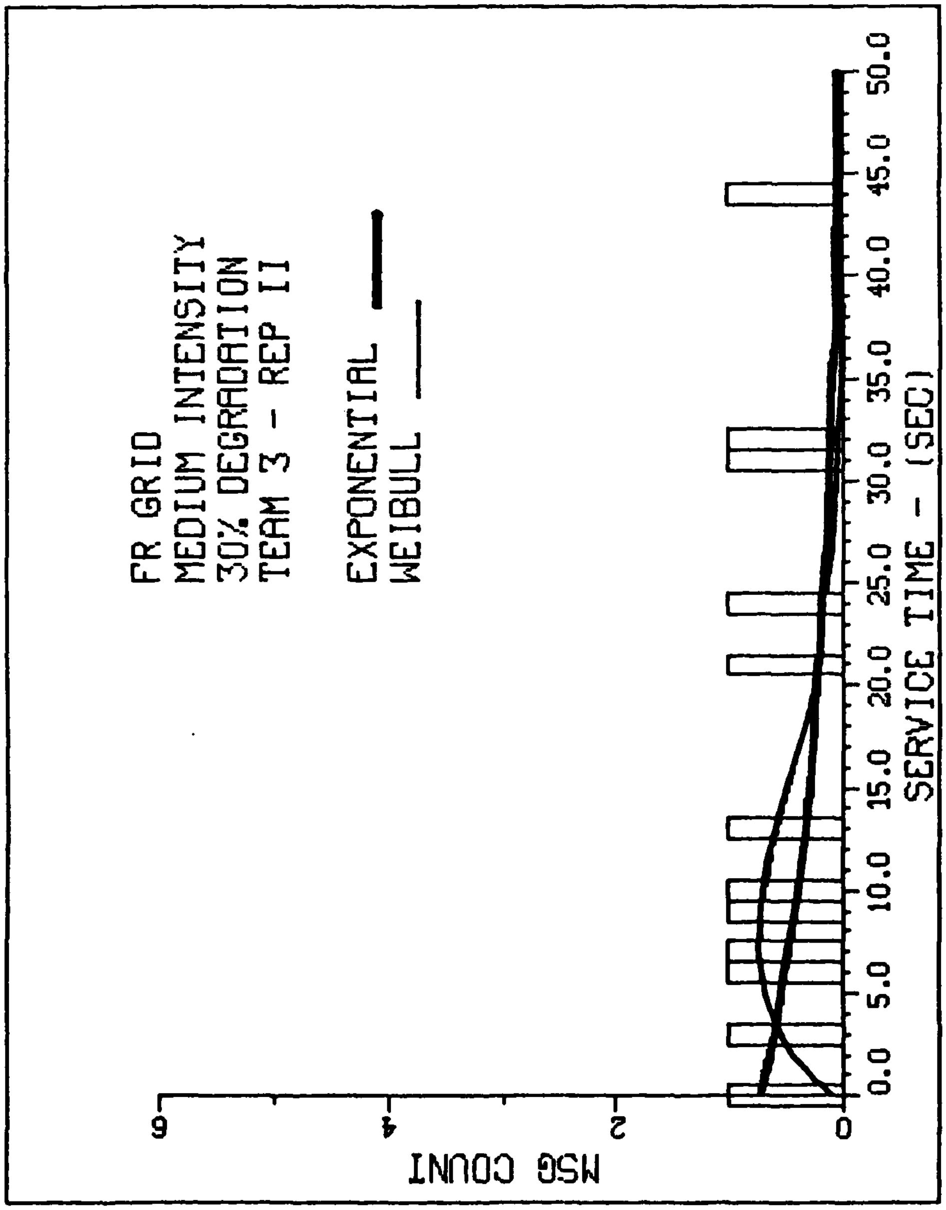
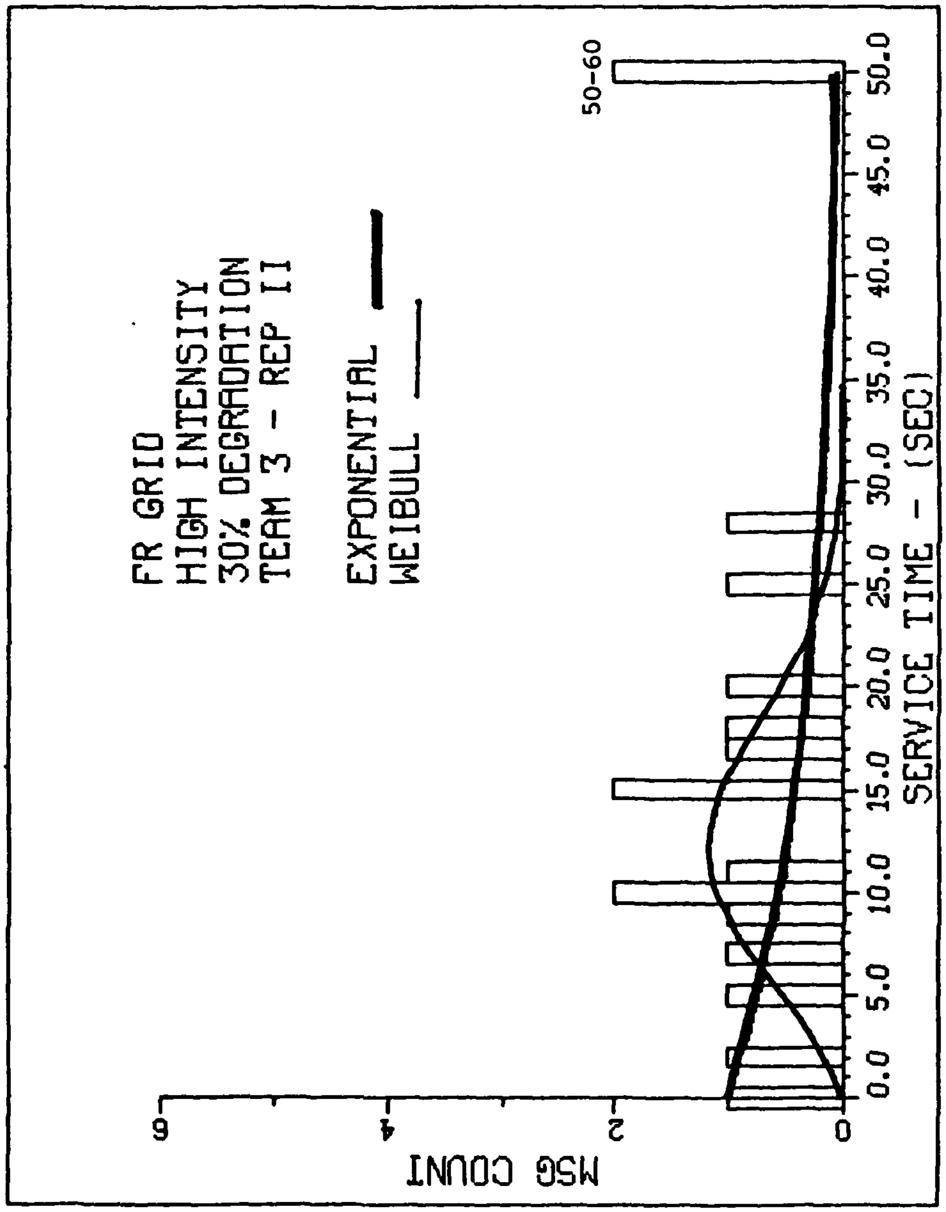
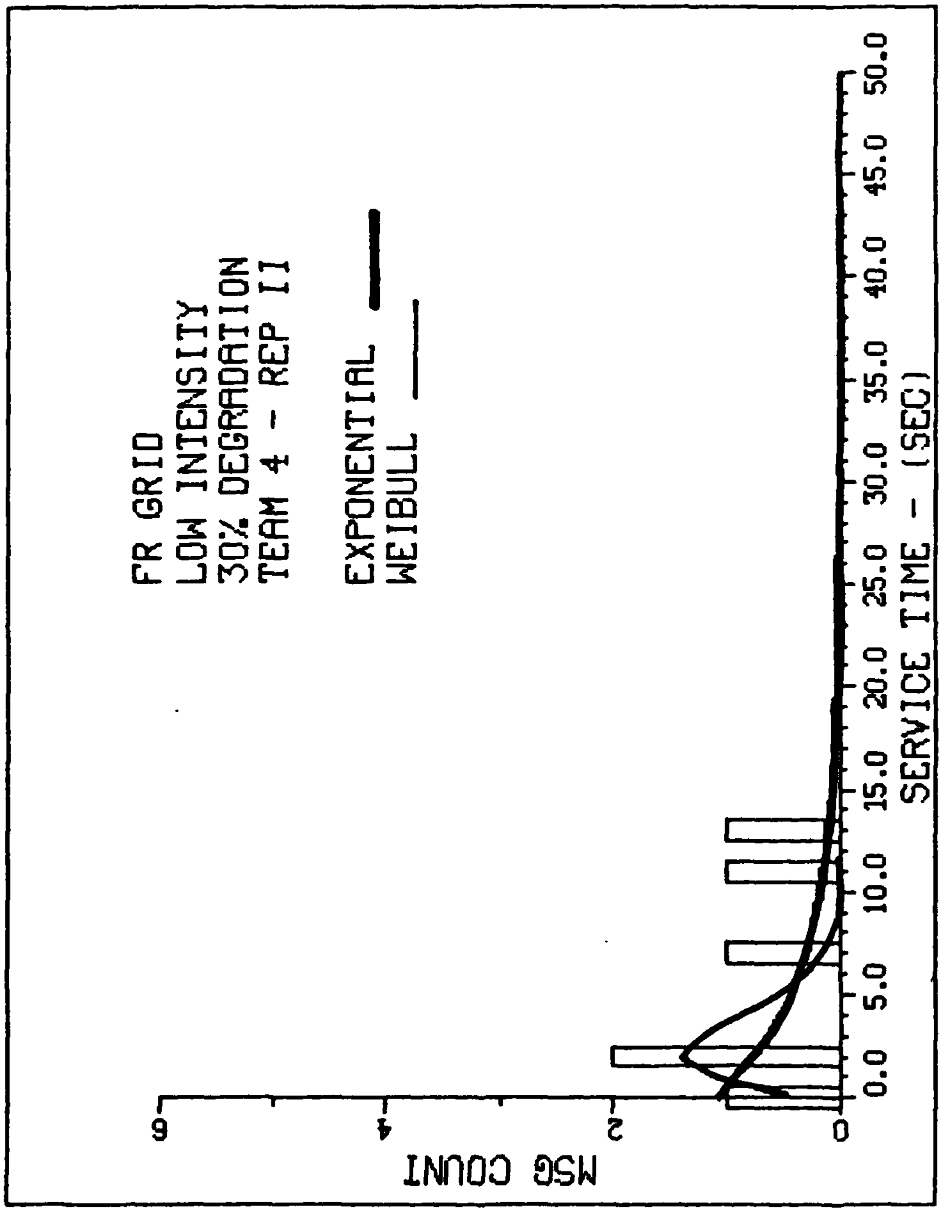
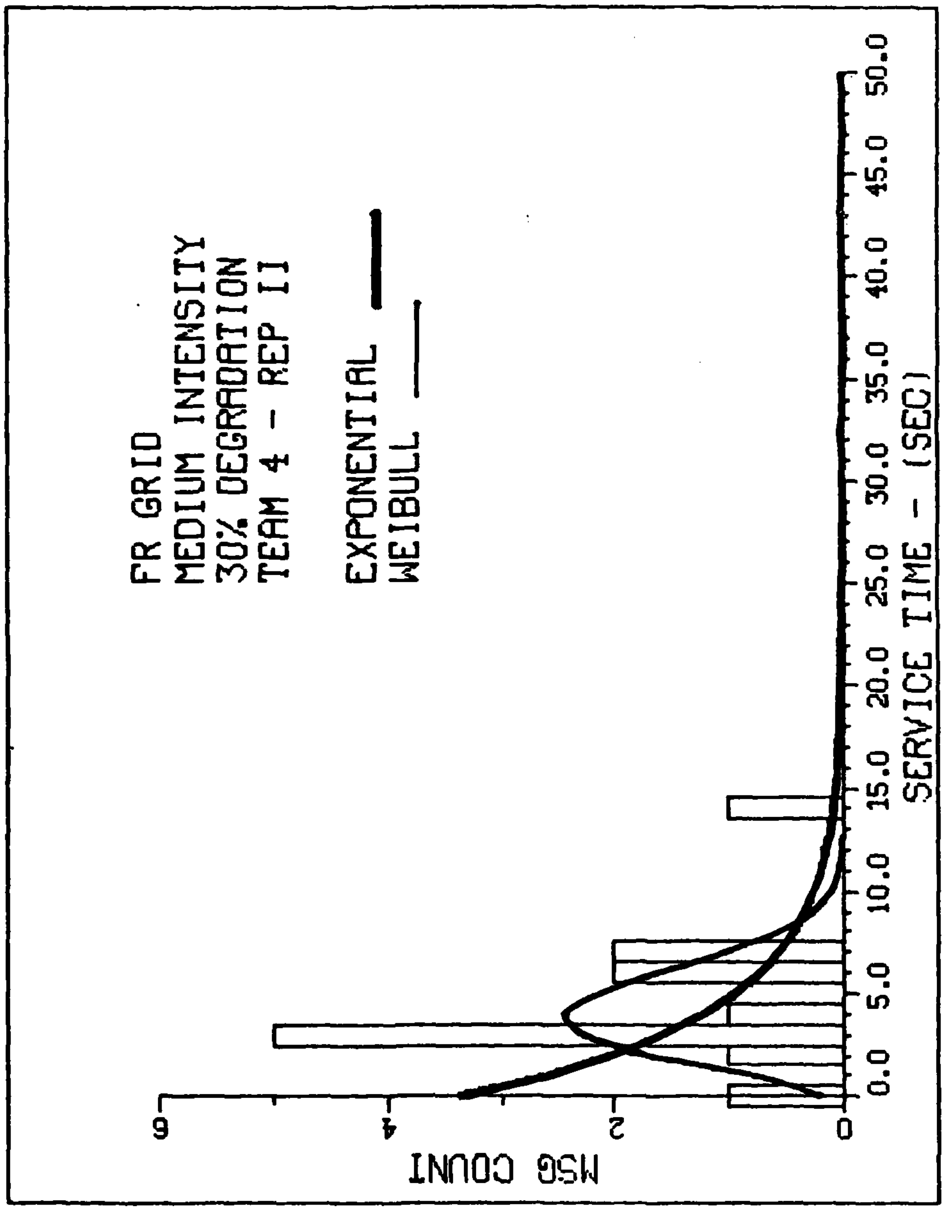
2
4
6
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
SERVICE TIME - (SEC)


FR GRID
LOW INTENSITY
30% DEGRADATION
TEAM 3 - REP II
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
SERVICE TIME - (SEC)

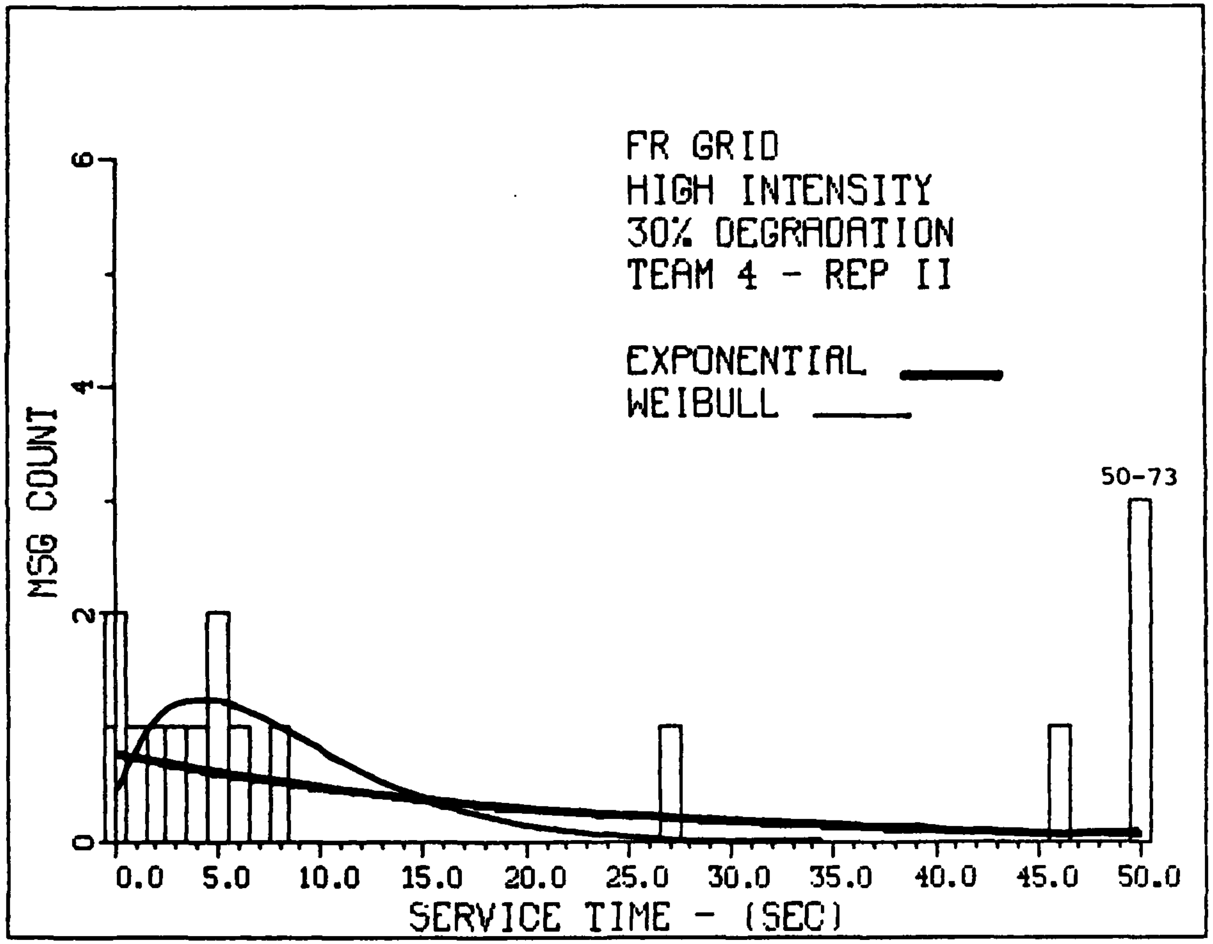

FR GRID
MEDIUM INTENSITY
30% DEGRADATION
TEAM 3 - REP II
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
SERVICE TIME - (SEC)


FR GRID
HIGH INTENSITY
30% DEGRADATION
TEAM 3 - REP II
EXPONENTIAL
WEIBULL
50-60
MSG COUNT
0
2
4
6
0.0
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
SERVICE TIME - (SEC)


FR GRID
LOW INTENSITY
30% DEGRADATION
TEAM 4 - REP II
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
SERVICE TIME - (SEC)



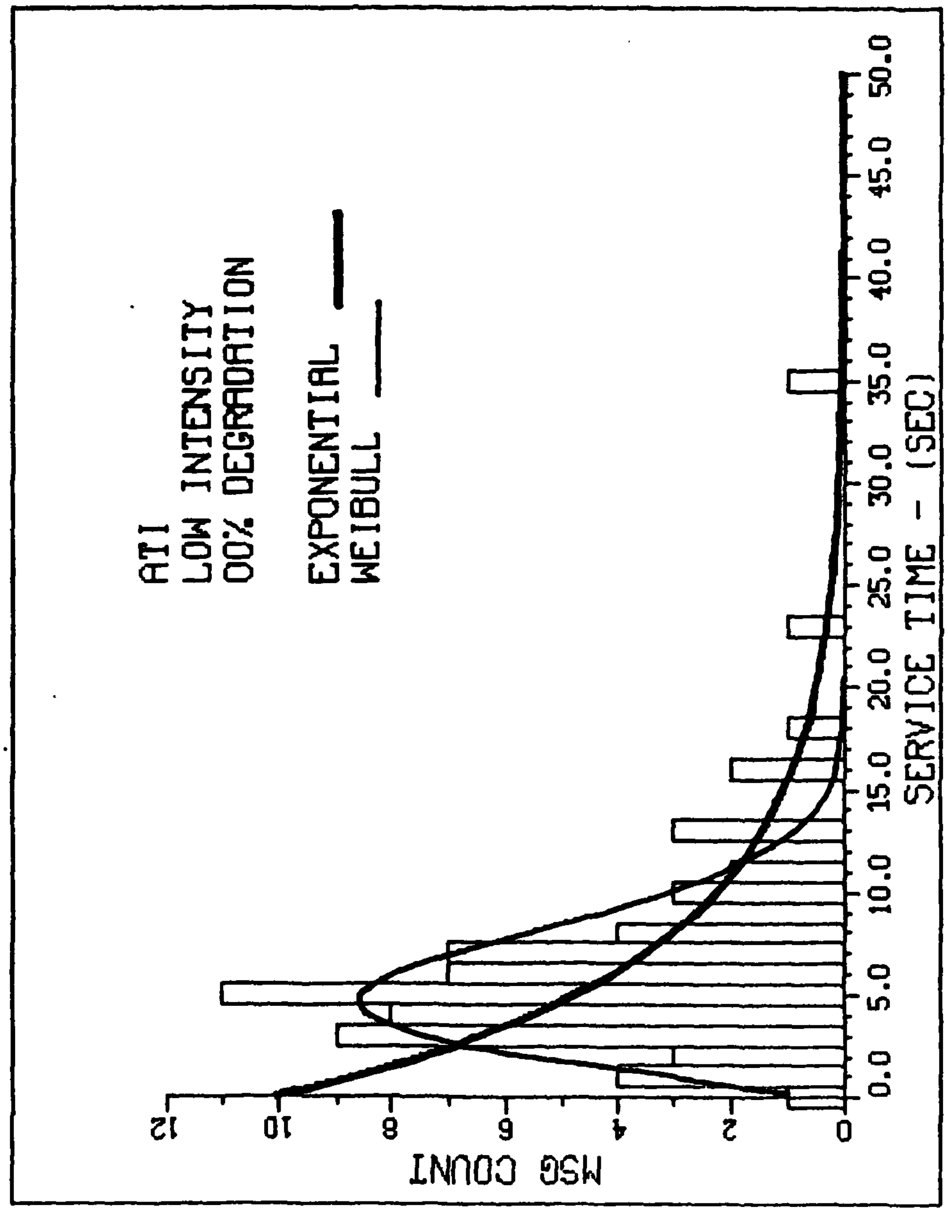
FR GRID
MEDIUM INTENSITY
30% DEGRADATION
TEAM 4 - REP II
EXPONENTIAL
WEIBULL
MSG COUNT
SERVICE TIME - (SEC)

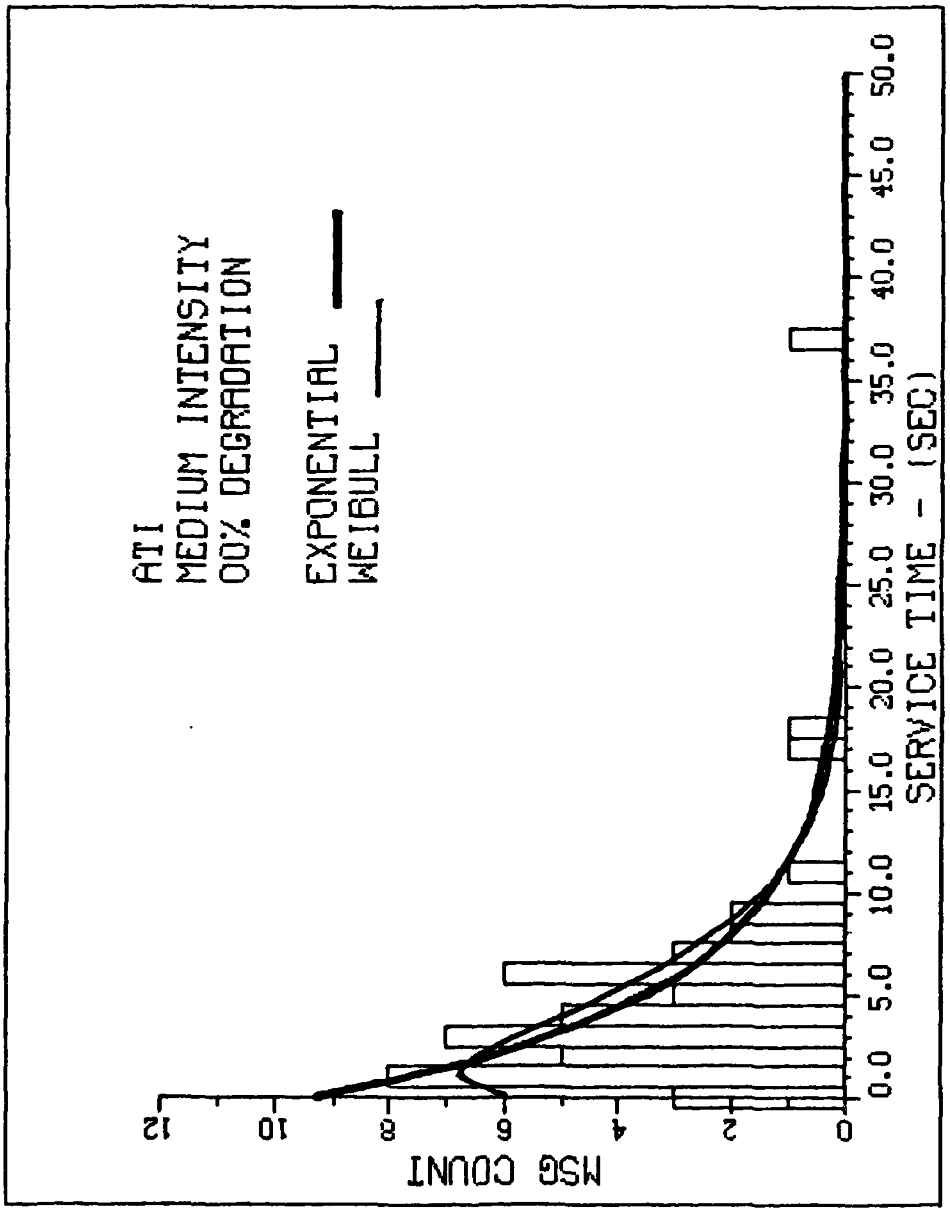


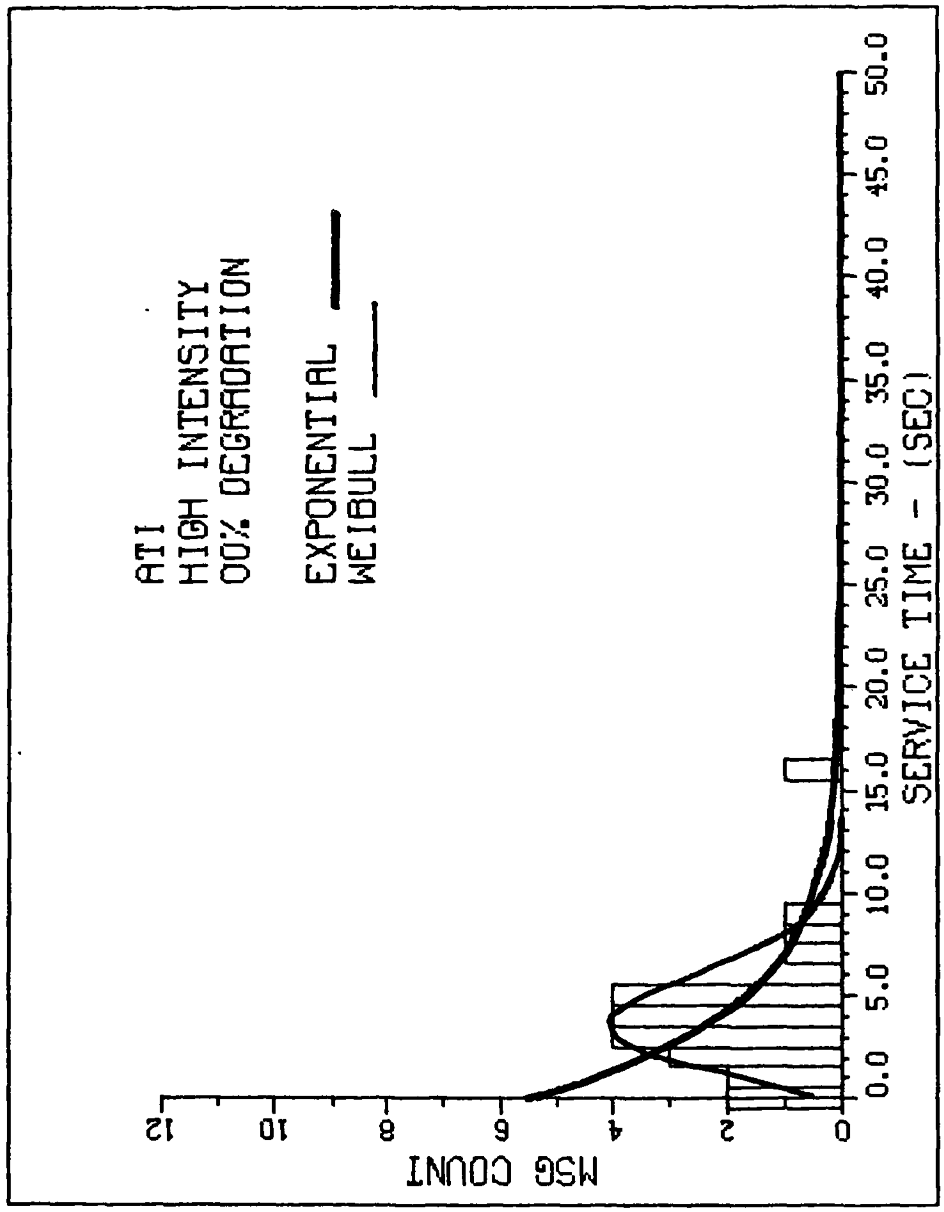
FR GRID
HIGH INTENSITY
30% DEGRADATION
TEAM 4 - REP II
EXPONENTIAL
WEIBULL
50-73
MSG COUNT
0
2
4
6
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
SERVICE TIME - (SEC)

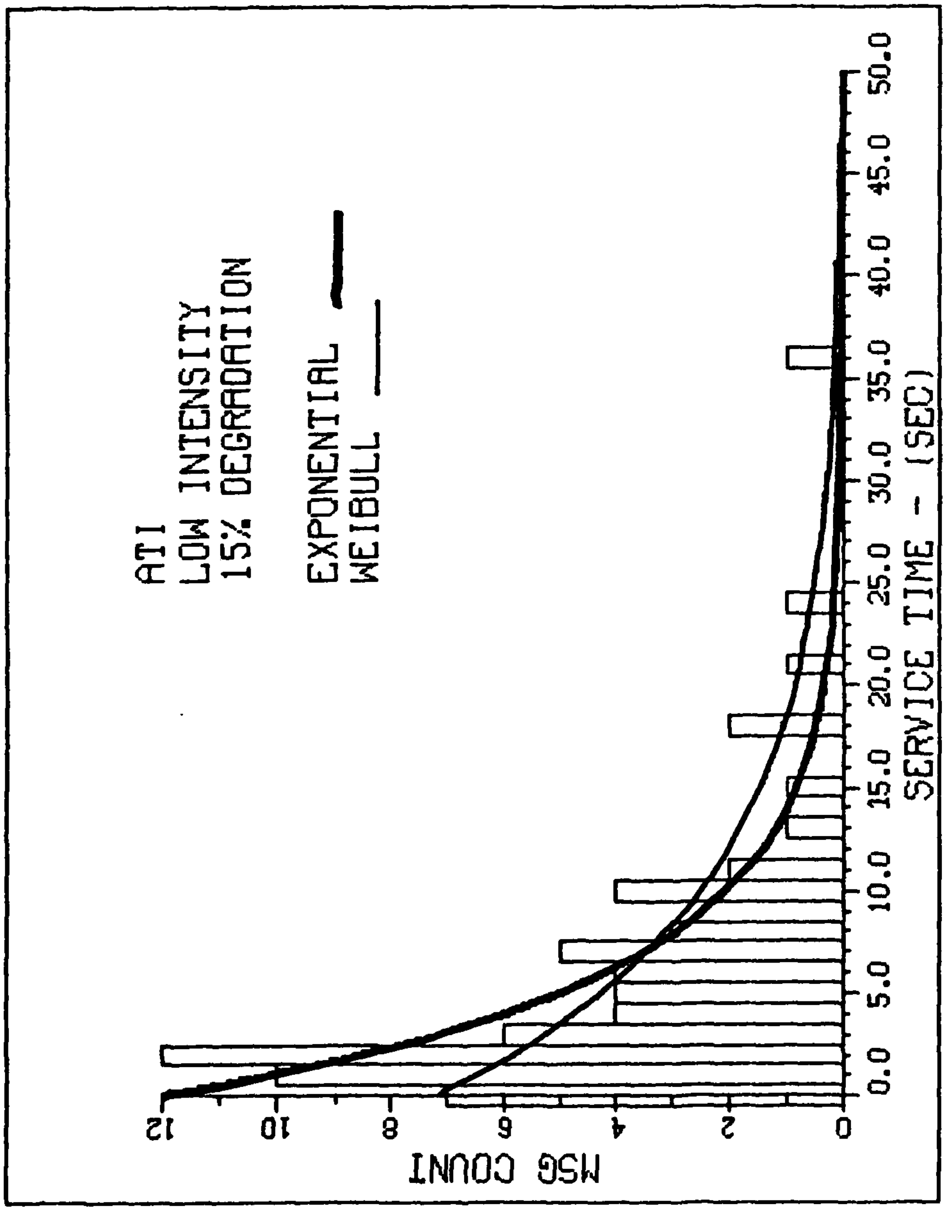
APPENDIX B.

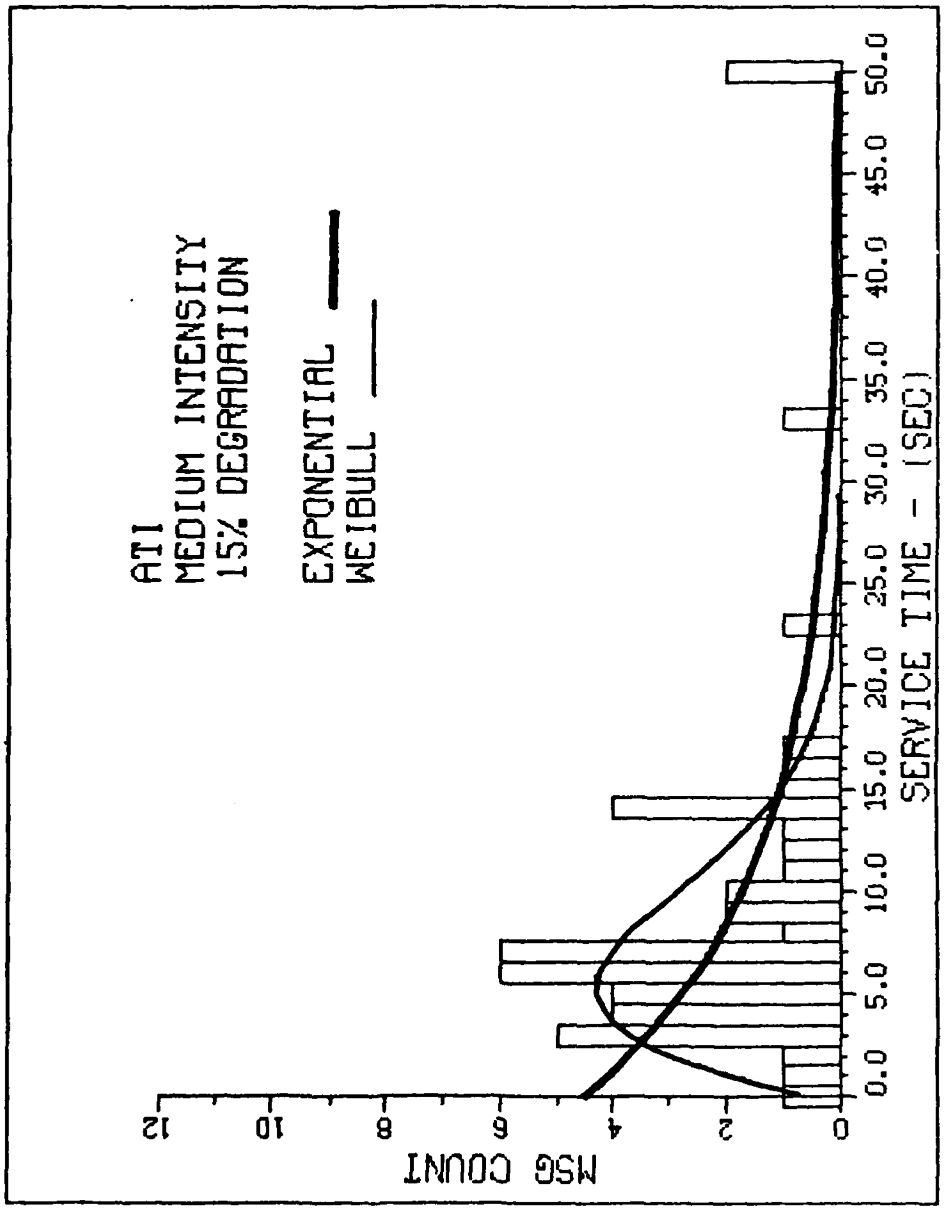
OBSERVED AND THEORETICAL FREQUENCY DISTRIBUTIONS FOR ARTILLERY TARGET INTELLIGENCE MESSAGES BY COMMUNICATION DEGRADATION AND INTENSITY LEVELS


ATI
LOW INTENSITY
00% DEGRADATION
EXPONENTIAL
WEIBULL
MSG COUNT
SERVICE TIME - (SEC)
0
2
4
6
8
10
12
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0

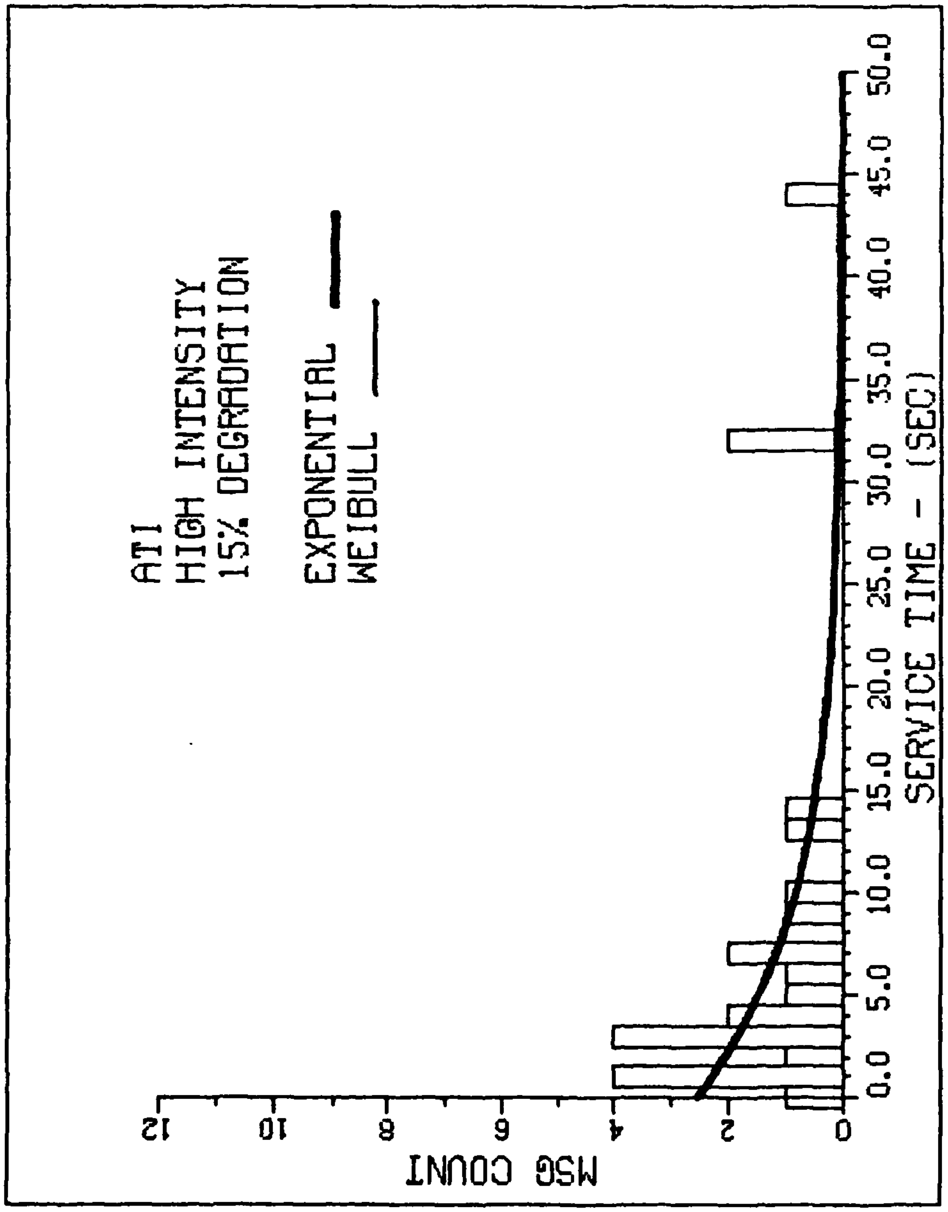

ATI
MEDIUM INTENSITY
00% DEGRADATION
EXPONENTIAL
WEIBULL
SERVICE TIME - (SEC)
MSG COUNT
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
0
2
4
6
8
10
12


ATI
HIGH INTENSITY
00% DEGRADATION
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
8
10
12
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
SERVICE TIME - (SEC)

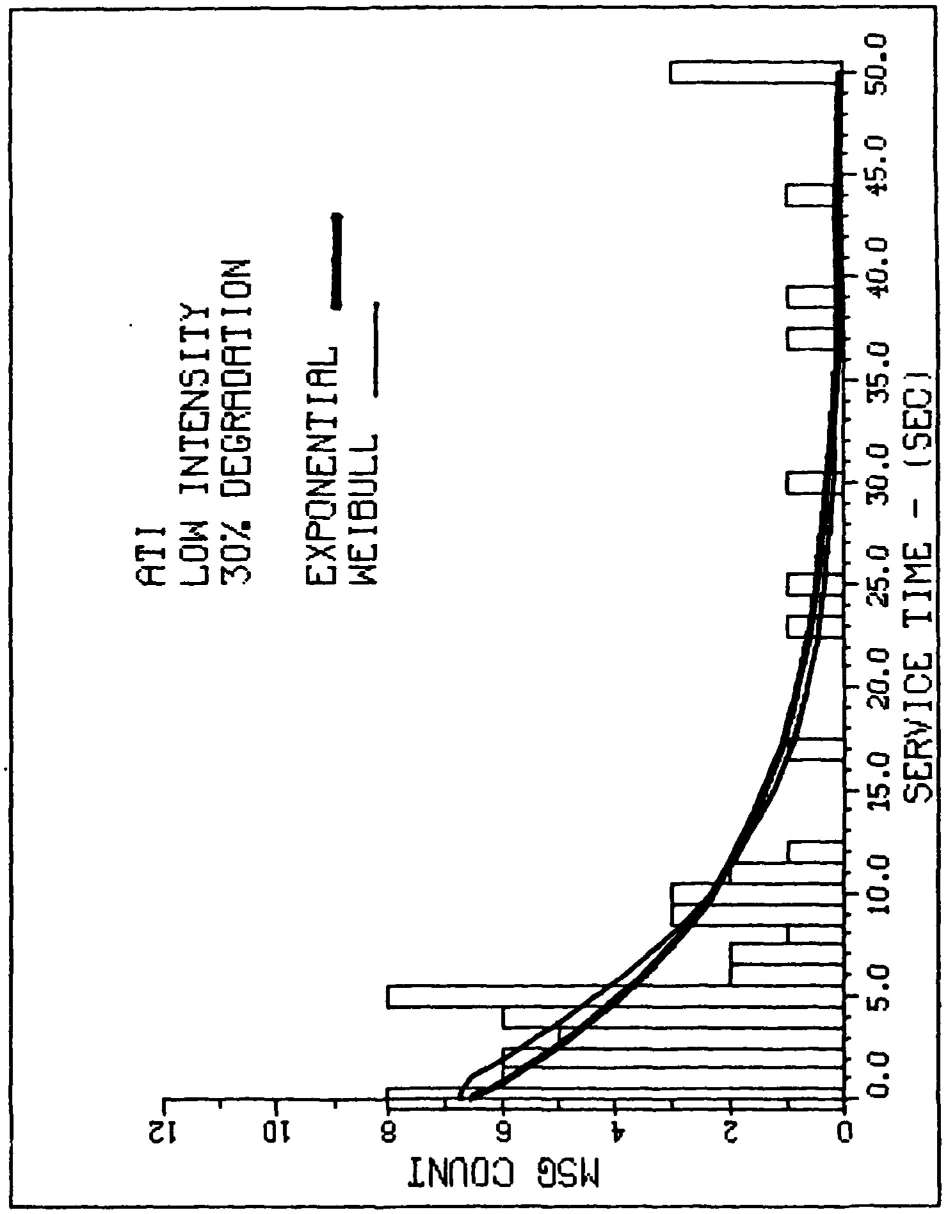

ATI
LOW INTENSITY
15% DEGRADATION
EXPONENTIAL
WEIBULL
MSG COUNT
SERVICE TIME - (SEC)
0
2
4
6
8
10
12
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0

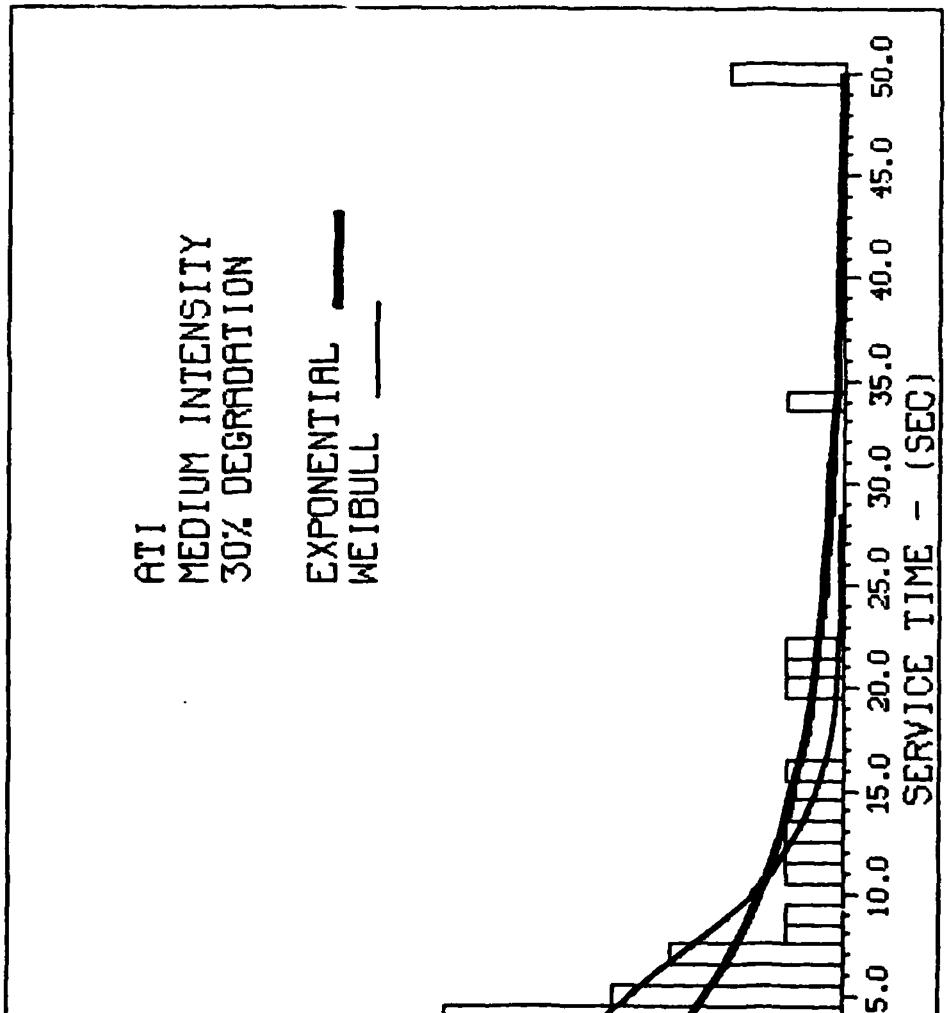


ATI
MEDIUM INTENSITY
15% DEGRADATION
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
8
10
12
SERVICE TIME - (SEC)
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0

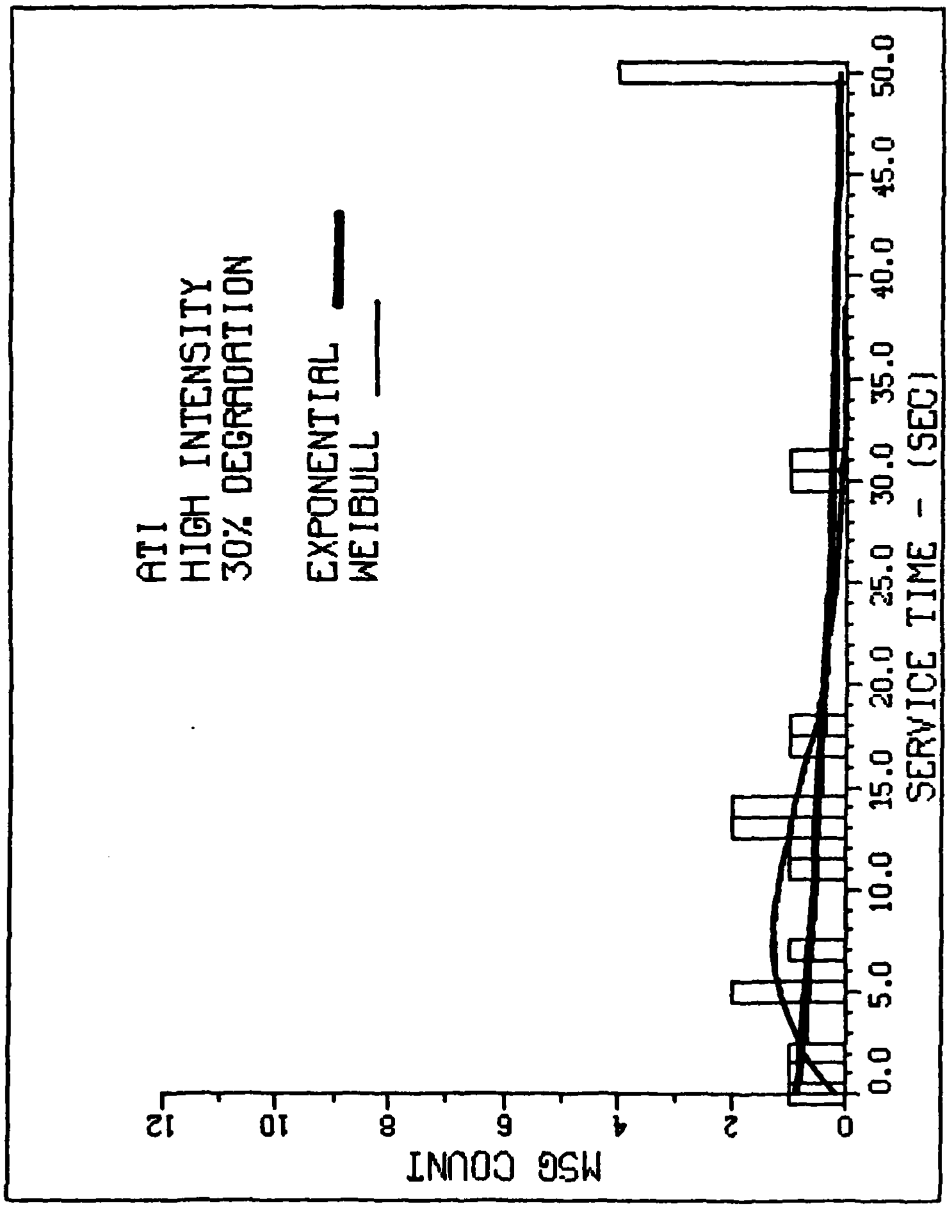


ATI
HIGH INTENSITY
15% DEGRADATION
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
8
10
12
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
SERVICE TIME - (SEC)


ATI
LOW INTENSITY
30% DEGRADATION
EXPONENTIAL
WEIBULL
SERVICE TIME - (SEC)
MSG COUNT
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
0
2
4
6
8
10
12



ATI
MEDIUM INTENSITY
30% DEGRADATION
EXPONENTIAL
WEIBULL
MSG COUNT
0
2
4
6
8
10
12
SERVICE TIME - (SEC)
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0


ATI
HIGH INTENSITY
30% DEGRADATION
EXPONENTIAL
WEIBULL
MSG COUNT
SERVICE TIME - (SEC)
0
2
4
6
8
10
12
0.0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0

APPENDIX C. TABLES

TABLE C-1A.

POINT ESTIMATES FOR THE 3-PARAMETER WEIBULL DISTRIBUTION FOR FIRE REQUEST MESSAGES (00% Degradation)

Rep	Team	Intensity	Parameters		
			α	β	ν
1	3	L	4.027	1.536	10.667
		M	4.708	1.176	10.889
		H	12.925	1.387	9.200
	4	L	4.274	1.620	8.500
		M	6.969	1.361	4.000
		H	8.197	1.203	4.000
2	3	L	10.345	2.040	7.500
		M	6.335	2.079	9.200
		H	14.444	2.128	3.500
	4	L	4.396	2.157	5.000
		M	5.319	1.016	8.000
		H	9.685	1.919	4.000

TABLE C-1B.

MAXIMUM LIKELIHOOD ESTIMATES FOR THE 3-PARAMETER WEIBULL DISTRIBUTION FOR FIRE REQUEST MESSAGES (00% Degradation)

Rep	Team	Intensity	Parameters		
			α	β	ν
1	3	L	8.610	6.447	11.000
		M	11.263	3.190	11.000
		H	8.488	2.450	11.000
	4	L	5.414	1.610	9.000
		M	1.050	1.175	5.000
		H	1.992	1.000	5.000
2	3	L	10.020	1.638	9.000
		M	7.299	1.926	10.000
		H	7.281	1.830	8.000
	4	L	3.399	1.691	6.000
		M	3.599	1.081	8.000
		H	6.903	1.717	6.000

TABLE C-2A.

POINT ESTIMATES FOR THE
3-PARAMETER WEIBULL DISTRIBUTION
FOR FIRE REQUEST MESSAGES
(15% Degradation)

Rep	Team	Intensity	Parameters		
			α	β	ν
1	3	L	19.481	1.158	12.500
		M	38.043	2.190	10.875
		H	25.404	2.030	2.000
	4	L	2.198	2.157	7.000
		M	8.264	.805	7.000
		H	7.519	.841	8.000
2	3	L	4.396	2.157	9.000
		M	5.282	1.098	9.875
		H	4.743	1.195	8.857
	4	L	5.859	1.247	7.500
		M	4.274	1.620	7.500
		H	6.579	1.023	5.750

TABLE C-2B.

MAXIMUM LIKELIHOOD ESTIMATES FOR THE
3-PARAMETER WEIBULL DISTRIBUTION
FOR FIRE REQUEST MESSAGES
(15% Degradation)

Rep	Team	Intensity	Parameters		
			α	β	ν
1	3	L	11.119	3.001	14.000
		M	7.561	3.674	14.000
		H	8.055	1.417	10.000
	4	L	3.364	1.740	7.000
		M	3.113	1.000	7.000
		H	6.927	1.000	8.000
2	3	L	4.450	3.512	11.000
		M	6.567	1.624	10.000
		H	7.351	1.000	9.000
	4	L	2.487	1.637	8.000
		M	4.195	1.735	8.000
		H	4.137	1.035	6.000

TABLE C-3A.

POINT ESTIMATES FOR THE 3-PARAMETER WEIBULL DISTRIBUTION FOR FIRE REQUEST MESSAGES (30% Degradation)

Rep	Team	Intensity	Parameters		
			α	β	ν
1	3	L	41.781	4.346	25.000
		M	9.284	0.787	11.923
		H	13.812	.995	10.500
	4	L	11.801	1.141	4.333
		M	13.043	2.187	1.000
		H	14.205	.976	3.500
2	3	L	21.127	.871	16.000
		M	23.018	1.809	5.000
		H	22.601	1.299	6.875
	4	L	7.627	2.049	3.667
		M	5.000	1.410	6.500
		H	8.632	.850	11.826

TABLE C-3B.

MAXIMUM LIKELIHOOD ESTIMATES FOR THE 3-PARAMETER WEIBULL DISTRIBUTION FOR FIRE REQUEST MESSAGES (30% Degradation)

Rep	Team	Intensity	Parameters		
			α	β	ν
1	3	L	35.391	8.844	25.000
		M	4.172	1.963	12.000
		H	12.443	1.210	11.000
	4	L	9.709	7.258	5.000
		M	5.400	2.647	5.000
		H	7.258	1.497	4.000
2	3	L	2.995	3.262	16.000
		M	12.905	1.650	8.000
		H	14.855	2.490	8.000
	4	L	3.304	1.707	5.000
		M	4.846	2.252	5.000
		H	8.967	1.475	12.000

TABLE C-4.

MAXIMUM LIKELIHOOD ESTIMATES OF θ FOR THE SINGLE PARAMETER EXPONENTIAL DISTRIBUTION FOR FIRE REQUEST MESSAGES (00% Degradation)

Rep	Team	Intensity	Parameter θ
1	3	L	.316
		M	.224
		H	.176
	4	L	.250
		M	.464
		H	.306
2	3	L	.109
		M	.302
		H	.247
	4	L	.462
		M	.271
		H	.250

TABLE C-5.

MAXIMUM LIKELIHOOD ESTIMATES OF θ FOR THE SINGLE PARAMETER EXPONENTIAL DISTRIBUTION FOR FIRE REQUEST MESSAGES (15% Degradation)

Rep	Team	Intensity	Parameter θ
1	3	L	.064
		M	.093
		H	.128
	4	L	.714
		M	.176
		H	.224
2	3	L	.417
		M	.200
		H	.173
	4	L	.194
		M	.520
		H	.247

TABLE C-6.

MAXIMUM LIKELIHOOD ESTIMATES OF θ FOR THE SINGLE PARAMETER EXPONENTIAL DISTRIBUTION FOR FIRE REQUEST MESSAGES
(30% Degradation)

Rep	Team	Intensity	Parameter θ
1	3	L	.046
		M	.090
		H	.072
	4	L	.115
		M	.128
		H	.107
2	3	L	.147
		M	.060
		H	.059
	4	L	.182
		M	.261
		H	.051

TABLE C-7.

COMPUTED KOLMOGOROV-SMIRNOV TEST STATISTIC VALUES
USING MAXIMUM LIKELIHOOD ESTIMATES FOR THE
SINGLE PARAMETER EXPONENTIAL DISTRIBUTION
FOR FIRE REQUEST MESSAGES
(00% Degradation)

Rep	Team	Intensity	Computed K-S Values (*D*)
1	3	L	.384
		M	.407*
		H	.322*
	4	L	.194
		M	.244
		H	.210
2	3	L	.230
		M	.365*
		H	.312*
	4	L	.269
		M	.231
		H	.316*

*Indicates the value of *D* is significant at a level of significance of .05.

TABLE C-8.

COMPUTED KOLMOGOROV-SMIRNOV TEST STATISTIC VALUES USING MAXIMUM LIKELIHOOD ESTIMATES FOR THE SINGLE PARAMETER EXPONENTIAL DISTRIBUTION FOR FIRE REQUEST MESSAGES (15% Degradation)

Rep	Team	Intensity	Computed K-S Values *(D)*
1	3	L	.243
		M	.205
		H	.214
	4	L	.400
		M	.369*
		H	.235
2	3	L	.411
		M	.292*
		H	.202
	4	L	.324
		M	.416*
		H	.113

*Indicates the value of *D* is significant at a level of significance of .05.

TABLE C-9.

COMPUTED KOLMOGOROV-SMIRNOV TEST STATISTIC VALUES
USING MAXIMUM LIKELIHOOD ESTIMATES FOR THE
SINGLE PARAMETER EXPONENTIAL DISTRIBUTION
FOR FIRE REQUEST MESSAGES
(30% Degradation)

Rep	Team	Intensity	Computed K-S Values *(D)*
1	3	L	.248
		M	.363*
		H	.169
	4	L	.185
		M	.278
		H	.171
2	3	L	.443*
		M	.136
		H	.179
	4	L	.220
		M	.508*
		H	.337*

*Indicates the value of *D* is significant at a level of significance of .05.

TABLE C-10A.

POINT ESTIMATES FOR THE 3-PARAMETER WEIBULL DISTRIBUTION FOR ATIs
(Degradation by Intensity)

Communication Degradation (%)	Intensity	Parameters		
		α	β	ν
00	L	11.050	1.669	2.000
	M	8.824	1.576	2.000
	H	3.521	.871	4.500
15	L	6.630	.995	4.800
	M	17.117	1.128	2.200
	H	11.180	.930	4.200
30	L	12.676	.871	3.200
	M	17.117	1.128	2.200
	H	29.206	1.102	4.750

TABLE C-10B.

MAXIMUM LIKELIHOOD ESTIMATES FOR THE 3-PARAMETER WEIBULL DISTRIBUTION FOR ATIs (Degradation by Intensity)

Communication Degradation (%)	Intensity	Parameters		
		α	β	ν
00	L	6.582	1.979	3.000
	M	5.266	1.185	4.000
	H	4.837	2.075	4.000
15	L	9.452	1.001	5.000
	M	8.606	1.725	3.000
	H	9.236	1.002	5.000
30	L	8.169	1.047	4.000
	M	6.369	1.372	4.000
	H	12.555	1.715	6.000

TABLE C-11.

MAXIMUM LIKELIHOOD ESTIMATES FOR THE SINGLE PARAMETER EXPONENTIAL DISTRIBUTION FOR ATIs
(Degradation by Intensity)

Communication Degradation (%)	Intensity	Parameter θ
	L	.150
00	M	.194
	H	.240
	L	.180
15	M	.096
	H	.112
	L	.104
30	M	.103
	H	.041

TABLE C-12.

COMPUTED χ^2 VALUES USING
MAXIMUM LIKELIHOOD ESTIMATES FOR THE
SINGLE PARAMETER EXPONENTIAL DISTRIBUTION
FOR ATIs

Communication Degradation (%)	Intensity	Computed χ^2 Values	Degrees of Freedom
00	L	24.354*	6
	M	7.672	5
	H	3.882	2
15	L	4.311	6
	M	12.579*	5
	H	.977	2
30	L	11.026	6
	M	10.731	5
	H	2.998	2

*Indicates a significant χ^2 value at the .05 level of significance.

APPENDIX D.

RELATIONSHIP BETWEEN THE WEIBULL AND EXPONENTIAL DISTRIBUTIONS

APPENDIX D.

RELATIONSHIP BETWEEN THE WEIBULL AND EXPONENTIAL DISTRIBUTIONS

Suppose that the random variable X, which represents the FIST HQ service time, has a three-parameter Weibull distribution, then the probability density function, $f_X(x)$ of X, has the following form:

$$f_X(x) = \begin{cases} \frac{\beta}{\alpha}\left(\frac{(x-\nu)}{\alpha}\right)^{\beta-1} \exp\left[-\left(\frac{x-\nu}{\alpha}\right)^{\beta}\right], & \text{if } x \geq \nu \\ 0, & \text{if } x < \nu;\ \beta, \alpha > 0; \\ & \nu \geq 0. \end{cases} \qquad \text{(D1)}$$

The three constants $\beta > 0$, $\alpha > 0$, and $\nu \geq 0$ are the parameters of the distribution. The parameter β determines the shape of the density function. The parameter α is a scale parameter specifying the $100\,[1 - \exp(-1)]$th distribution percentile of $X - \nu$. The parameter ν will assume the smallest possible value of the random variable X. It may be thought of as a location (or threshold) parameter where a message will be serviced before time ν with probability 0.

Weibull random variables may be easily transformed to exponentially distributed random variables. An exponential model can be utilized if an appropriate transformation of the FIST HQ service time data is made. Suppose ordered service times $x_{(1)}, \cdots, x_{(r)}$ from a sample of size n are observed and the observations are from a population of Weibull random variables with known shape and location parameters β and ν, respectively, and unknown scale parameter α. Then $y_{(1)} = (x_{(1)} - \nu)^{\beta}, \cdots, y_{(r)} = (x_{(r)} - \nu)^{\beta}$ will be considered ordered observations from an exponential distribution with unknown scale parameter $\theta = \left(\frac{1}{\alpha}\right)^{\beta}$.

This transformation is based on the assumption that β and ν are assumed to be known for a given set of treatment combinations. Based on the maximum likelihood estimates for β and ν (see Tables C-1B, C-2B, C-3B and C-10B), the value for β will be set equal to 1.000, while ν will be set equal to the smallest order statistic of each data set. Thus, the unknown scale parameter of the exponential distribution becomes $\theta = \frac{1}{\alpha}$.

To verify the assertion that the Weibull distribution is equivalent to the exponential distribution with parameter $\theta = \frac{1}{\alpha}$ under the above assumptions, the ordered observations from the exponential distribution $y_{(1)} = (x_{(1)} - \nu)^{\beta}, \ldots, y_{(r)} = (x_{(r)} - \nu)^{\beta}$ can be rewritten as $y_{(1)} = (x_{(1)} - \nu), \ldots, y_{(r)} = (x_{(r)} - \nu)$.

Thus $Y = [\,(X - \nu)\,] = g(X)$ is a general transformation of the continuous random variable X; we would like to obtain the distribution of Y from the distribution of X. Since the observations $y_{(1)}, \ldots, y_{(r)}$ are from the exponential distribution, then $Y = g(X)$ is a strictly decreasing, differentiable transformation. A strictly decreasing

transformation of the form $y = g(x)$ implies that if $g(x_1) > g(x_2)$ then $x_1 < x_2$. Differentiable implies $(d/dx)g(x)$ exists for every x. If $y = g(x) = (x - \nu)$ is strictly decreasing and differentiable, there exists a decreasing, differentiable function h(y) such that $x = h(y) = y + \nu$.

Thus, the density function of $Y = g(X)$ becomes,

$$F_Y(y) = P\{Y \leq y\} = P\{g(X) \leq y\} = P\{X \geq h(y)\} = 1 - F_X(h(y)). \quad \text{(D2)}$$

Substituting $g(X) = (X - \nu)$ and $h(y) = y + \nu$, equation D2 becomes,

$$F_Y(y) = P\{Y \leq y\} = P\{(X - \nu) \leq y\} = P\{X \geq (y + \nu)\} \quad \text{(D3)}$$
$$= 1 - F_X(y + \nu).$$

If both sides of the equality are differentiated with respect to y, the probability density function of Y is

$$f_Y(y) = \frac{dh(y)}{dy}\left(-f_X(h(y))\right) = \left|\frac{dh(y)}{dy}\right| f_X(h(y)), \quad \text{(D4)}$$

since $(d/dy)\,h(y) < 0$.

Substituting for h(y) gives

$$f_Y(y) = \frac{d(y+\nu)}{dy}\left(-f_X(y+\nu)\right) = \left|\frac{d(y+\nu)}{dy}\right| f_X(y+\nu). \quad \text{(D5)}$$

Hence, the probability density function of Y is

$$f_Y(y) = \left|\frac{d(y+\nu)}{dy}\right| \left[\frac{\beta}{\alpha}\left(\frac{(y+\nu)-\nu}{\alpha}\right)^{\beta-1} \exp\left(-\left(\frac{(y+\nu)-\nu}{\alpha}\right)^{\beta}\right)\right] \quad \text{(D6)}$$

Recall that $\beta = 1.000$, then

$$f_Y(y) = \left|\frac{d(y+\nu)}{dy}\right| \frac{1}{\alpha}\exp\left(-\left(\frac{y}{\alpha}\right)\right), \quad \text{(D7)}$$

thus

$$f_Y(y) = \frac{1}{\alpha}\exp\left(-\left(\frac{y}{\alpha}\right)\right). \quad \text{(D8)}$$

It was previously stated that $\theta = \frac{1}{\alpha}$, therefore, substituting in equation D8 gives

$$f_Y(y) = \theta \exp(-(\theta y)). \quad \text{(D9)}$$

Therefore, the distributional form of the transformed random variables $y_{(1)}, \ldots, y_{(r)}$ is exponential with unknown scale parameter θ, while the original observations were considered to be from a population of Weibull random variables.

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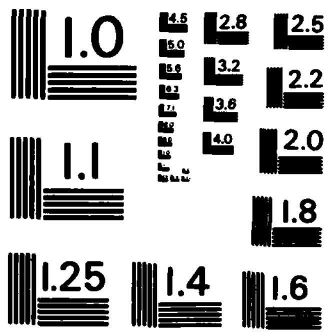

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